§1.1.

P2. $y = 3e^{-2x}$

$y' = -6e^{-2x}$

$\Rightarrow y' + 2y = -6e^{-2x} + 6e^{-2x} = 0$

P3. $y_1 = \cos 2x$

$y_1' = -2\sin 2x$, $y_1'' = -4\cos 2x$

$\Rightarrow y_1'' + 4y_1 = -4\cos 2x + 4\cos 2x = 0$

$y_2 = \sin 2x$

$y_2' = 2\cos 2x$, $y_2'' = -4\sin 2x$

$\Rightarrow y_2'' + 4y_2 = -4\sin 2x + 4\sin 2x = 0$

P7. $y_1 = e^x \cos x$

$y_1' = -e^x \sin x + e^x \cos x$

$y_1'' = -e^x \sin x - \cancel{e^x \cos x} + \cancel{e^x \cos x} - e^x \sin x$

$= -2e^x \sin x$

$\Rightarrow y_1'' - 2y_1' + 2y_1 = -2e^x \sin x - 2(-e^x \sin x + e^x \cos x)$
$+ 2e^x \cos x$
$= 0$

$y_2 = e^x \sin x$

$y_2' = e^x \sin x + e^x \cos x$

$y_2'' = e^x \sin x + e^x \cos x + e^x \cos x - e^x \sin x$

$= 2e^x \cos x$

$\Rightarrow y_2'' - 2y_2' + 2y_2 = 2e^x \cos x - 2(e^x \sin x + e^x \cos x)$
$+ 2e^x \sin x$
$= 0$

P10. $y_1 = x - \ln x$

$y_1' = 1 - \frac{1}{x}$, $y_1'' = \frac{1}{x^2}$

$\Rightarrow x^2 y_1'' + x y_1' - y_1 = x^2 \cdot \left(\frac{1}{x^2}\right) + x \cdot \left(1 - \frac{1}{x}\right) - (x - \ln x)$
$= \ln x$

$y_2 = \frac{1}{x} - \ln x$

$y_2' = -\frac{1}{x^2} - \frac{1}{x}$, $y_2'' = \frac{2}{x^3} + \frac{1}{x^2}$

$x^2 y_2'' + x y_2' - y_2 = x^2\left(\frac{2}{x^3} + \frac{1}{x^2}\right) + x\left(-\frac{1}{x^2} - \frac{1}{x}\right)$
$- \left(\frac{1}{x} - \ln x\right)$
$= \ln x$

P17. Solution:

$y(x) = Ce^{-x}$. $y'(x) = -Ce^{-x}$

$y' + y = -Ce^{-x} + Ce^{-x} = 0$

$y(0) = 2 \Rightarrow C \cdot e^{-0} = 2 \Rightarrow C = 2$

$\Rightarrow y(x) = 2e^{-x}$

is the solution of the initial value problem.

P22. Solution:

$y(x) = \ln(x + c)$, $y'(x) = \frac{1}{x+c}$

$e^y y' = e^{\ln(x+c)} \cdot \frac{1}{x+c} = 1$

$y(0) = 0 \Rightarrow \ln(0 + c) = 1 \Rightarrow c = 1$

$\Rightarrow y(x) = \ln(x+1)$

is the solution of the initial value problem.

P27. Solution:

$$\frac{dy}{dx} = x + y$$

P28. solution:

$$\frac{dy}{dx} = \frac{y - 0}{x - x/2} = \frac{y}{x/2} = \frac{2y}{x}$$

P32. Solution:

$$\frac{dP}{dt} = k\sqrt{P}$$

where k is a constant.

P43. Solution:

(a) $x(t)=\frac{1}{c-kt}$.

$x'=\frac{k}{(c-kt)^2}=kx^2$

(b). $x(0)=0$

$\Rightarrow \frac{1}{c-kt}=0 \Rightarrow \frac{1}{c}=0$

$c=\infty$

$\Rightarrow x\equiv 0$

§1.2.

P1. Solution:

$y(x)=\int 2x+1\,dx+C$

$=x^2+x+C$

$y(0)=3 \Rightarrow C=3$

$\Rightarrow y(x)=x^2+x+3$

P2. Solution.

$y(x)=\int (x-2)^2 dx+C$

$=\frac{1}{3}(x-2)^3+C$

$y(2)=1 \Rightarrow C=1$

$\Rightarrow y(x)=\frac{1}{3}(x-2)^3+1$

P5. Solution:

$y(x)=\int \frac{1}{\sqrt{x+2}}dx \overset{+C}{=} 2\sqrt{x+2}+C$

$y(2)=-1 \Rightarrow 2\cdot\sqrt{4}+C=-1$

$\Rightarrow C=-5$

$\Rightarrow y(x)=2\sqrt{x+2}-5$

P8. Solution: $y(x)=\int \cos 2x\,dx+C$

$=\frac{1}{2}\sin 2x+C$

$y(0)=1 \Rightarrow C=1$

$\Rightarrow y(x)=\frac{1}{2}\sin 2x+1$

P10. solution: $y(x)=\int xe^{-x}dx+C$

$=x(-e^{-x})-\int(-e^{-x})dx+C$

$=-xe^{-x}-e^{-x}+C$

$y(0)=1 \Rightarrow -1+C=1 \Rightarrow C=2$

$\Rightarrow y(x)=e^{-x}(-x-1)+2$

P11. Solution: $a(t)=50$, $V_0=10$, $X_0=20$

$x(t)=\frac{1}{2}at^2+V_0t+X_0$

$=25t^2+10t+20$.

P13. solution: $a(t)=3t$; $V_0=-15$, $X_0=5$

$V(t)=\int_0^t a(s)ds+V_0=\int_0^t 3s\,ds+(-15)$

$=\frac{3}{2}s^2\Big|_{s=0}^{s=t}-15=\frac{3}{2}t^2-15$

$X(t)=\int_0^t V(s)ds+X_0=\int_0^t(\frac{3}{2}s^2-15)ds+5$

$=(\frac{1}{2}s^3-15s)\Big|_{s=0}^{s=t}+5$

$=\frac{1}{2}t^3-15t+5$

P16. solution: $a(t)=\frac{1}{\sqrt{t+4}}$, $V_0=-1$, $X_0=1$

$V(t)=\int a(t)dt+C_1=\int\frac{1}{\sqrt{t+4}}dt+C_1$

$=2\sqrt{t+4}+C_1$

$V(0)=-1 \Rightarrow 4+C_1=-1 \Rightarrow C_1=-5$

$\Rightarrow V(t)=2\sqrt{t+4}-5$

$X(t)=\int V(t)dt+C_2=\int 2\sqrt{t+4}-5\,dt+C_2$

$=\frac{4}{3}(t+4)^{\frac{3}{2}}-5t+C_2$

$X(0)=1 \Rightarrow \frac{4}{3}\cdot 8+C_2=1 \Rightarrow C_2=-\frac{29}{3}$

$\Rightarrow X(t)=\frac{4}{3}(t+4)^{3/2}-5t-\frac{29}{3}$

P25. Solution: $X(t)=\frac{1}{2}at^2+V_0t+X_0$, $V(t)=at+V_0$

Notice $X_0=X(0)=0$, $V_0=V(0)=100000m/h$

$a=-10m/s$ $\quad =250/9\ m/s$

$\Rightarrow X(t)=-5t^2+\frac{250}{9}t$

$V(t)=-10t+\frac{250}{9}$

To reach a stop, we have $V(t)=0$

$\Rightarrow -10t+\frac{250}{9}=0 \Rightarrow t=\frac{25}{9}$

$\Rightarrow X(\frac{25}{9})=-5\cdot(\frac{25}{9})^2+\frac{250}{9}\cdot\frac{25}{9}\approx 38.58m$

P28. solution:

$V_0 = -40$ ft/s, $g = 32$ ft/s^2, $X_0 = 555$ ft

$X(t) = -\frac{1}{2} g t^2 + V_0 t + X_0$

$= -16t^2 - 40t + 555$

The baseball reaches the ground, i.e.

$X(t) = 0$

$\Rightarrow -16t^2 - 40t + 555 = 0$

$t = \dfrac{40 \pm \sqrt{40^2 - 4 \times (-16) \times 555}}{2 \cdot (-16)}$

$\Rightarrow t_1 \approx -7.27s$, $\boxed{t_2 \approx 4.77s}$

The time has to be positive, thus it take about 4.77s.

$V(t) = -gt + V_0$

$= -32t - 40$

$V(4.77) = (-32) \cdot (4.77) - 40$

≈ -192.64 ft/s