Name: MATH 241 §004

Final Exam

Complete the following problems to the best of your ability. **SHOW ALL OF YOUR WORK.** Unshown work will not be graded. You may not use a calculator.

1. Let $\mathbf{u} = \langle 2, 1 \rangle$, and $\mathbf{v} = \langle 0, 5 \rangle$. Calculate the following.

   (a) $3\mathbf{u} - 2\mathbf{v}$

   (b) $|\mathbf{v}|\mathbf{u}$

   (c) $\text{proj}_{\mathbf{v}} \mathbf{u}$

2. Let $l_1 = \langle 2t - 1, 1 + t, 3t \rangle$ and $l_2 = \langle -t, 3 + t, -2 - 5t \rangle$. Let $S = (0, 1, 3)$.

   (a) Calculate the intersection between l_1 and l_2, or explain why it doesn't exist.

   (b) Calculate the distance between l_1 and S.

3. Let $P = (0, 1, 0)$, $Q = (2, 1, 1)$, and $R = (-1, 3, 0)$.

(a) Calculate the distance between P and Q.

(b) Give a parameterisation of the line between P and Q.

(c) Calculate the area of the triangle $\triangle PQR$.

(d) Find the equation of the plane containing P, Q and R.

4. Horcy is back at it again. This time, his ship is taking the path traced by

$$\mathbf{r}(t) = \langle 2 + 3\sin t, 1 - 4\sin t, 7 + 5\cos t \rangle,$$

where t is measured in seconds.

(a) What is Horcy's speed after π seconds?

(b) Give a parameterisation of the line tangent to $\mathbf{r}(t)$ at $t = \pi/2$.

(c) What is the total distance travelled by Horcy in the first 2π seconds of his journey?

5. Let $f(x,y) = x^3+x^2+xy^2$. Find all saddle points and global extrema of f on the disc $\{(x,y)|x^2+y^2 \leq 4\}$. Include the z-coordinates of these points, as well as the x and y.

6. Switch the order of integration and calculate the value of the integral

$$\int_0^1 \int_{x^2}^1 e^{y^{3/2}}\, dy\, dx$$

7. Convert the following integral into polar coordinates, then evaluate it.

$$\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (x^2+y^2)^3\, dy\, dx$$

8. Use a triple integral to calculate the volume in the first octant between the spheres $\rho = 1$ and $\rho = 2$.

9. Rewrite the triple integral below in cylindrical coordinates. You do not need to evaluate the integral.

$$\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} \int_{0}^{1-x^2-y^2} z(x^2+y^2+z^2)^{3/2}\, dz\, dy\, dx$$

10. Use Greene's Theorem to calculate

$$\oint_C (y - ye^x + 2x\cos y)\,dx + (3x - e^x - x^2\sin y)\,dy$$

where C is the anti-clockwise curve consisting of the line segment from $(-2, 0)$ to $(2, 0)$ and the portion of the circle $x^2 + y^2 = 4$ above the x-axis beginning at $(2, 0)$ and ending at $(-2, 0)$.