Test 2: Ch. 13-14

Complete the following problems to the best of your ability. **SHOW ALL OF YOUR WORK.**
Unshown work will not be graded. You may not use a calculator.

1. Horcy is flying his subatomic spaceship through the coordinate plane. His trajectory is traced by the vector function $\mathbf{r}(t) = \langle 2t, t^2, \frac{t^3}{3} \rangle$, where t is measured in seconds.

   (a) Calculate $|\mathbf{r}'(3)|$. Give a real-world interpretation of this value.

   (b) Give a parameterisation of the tangent line to $\mathbf{r}(t)$ at $t = 3$.

   (c) What is the total distance travelled by Horcy from $t = 1$ to $t = 2$?

   (d) Horcy's nemesis, Donkey Hotay, is trying to ram his ship with a ship of his own. Hotay's ship follows the path given by $\mathbf{q}(t) = \langle t, \sin(t), \cos(t) \rangle$. Do the ships collide?

2. The following limit does not exist. Prove why, using a rigorous argument.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^4 + y^4 + 2xy^3}}$$

3. Find the equation of the plane tangent to the surface $x^4 + xy^2 - y^2z = 1$ at the point $(1,1,1)$.

4. Let $f(x,y) = xy^2 + \sin(xy)$.

   (a) In which direction is the directional derivative of f maximal at $(0,1)$?

   (b) What is the directional derivative of f in the direction of $\mathbf{v} = \langle 1,3 \rangle$ at the point $P = (\pi, 0)$?

5. Let $f(x, y) = 2x^2 + y^2 - 2x - 3$.

   (a) Find all local extrema and saddle points of f. Include the z-coordinates, not just the x and y.

   (b) Find all global extrema of f within the triangle bounded by $x = 0$, $y = 1$, and $y = x$. Again, include z-coordinates as well as x and y.