Name: MATH 241 §004

Test 1: Ch. 12

Complete the following problems to the best of your ability. **SHOW ALL OF YOUR WORK.** Unshown work will not be graded. You may not use a calculator.

1. [15] Mark the following as true (T) or false (F). If they are false, explain why or give a counterexample.

   _____ A unit vector is a vector of magnitude one.

   _____ Two vectors are orthogonal if their cross product is zero.

   _____ The projection $\text{proj}_{\mathbf{v}}\,\mathbf{u}$ is orthogonal to the vector $\mathbf{v}$.

   _____ The cross product $\mathbf{u} \times \mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$, as long as $\mathbf{u}$ and $\mathbf{v}$ aren't parallel.

   _____ Two lines are parallel if their direction vectors are scalar multiples of each other.

2. [8] Let $\mathbf{u} = \langle 4, 6 \rangle$ and $\mathbf{v} = \langle -1, 2 \rangle$. Calculate the following.

   (a) $\mathbf{u} - 3\mathbf{v}$

   (b) $\text{proj}_{\mathbf{v}}\,\mathbf{u}$

3. [40] Let P be the point $(0, 2, -1)$, $Q = (1, -1, 1)$ and $R = (2, 0, 1)$.

(a) Calculate the distance between P and Q.

(b) Give a paramaterisation of the line betwen P and R.

(c) Calculate the area of the triangle $\triangle PQR$.

(d) Find the equation of the plane containing P, Q and R.

4. [15] Find the point of intersection between the following two lines.

$$\begin{aligned} l_1: &\quad x = 2 - t, && y = 1 + t, && z = 2 + t \\ l_2: &\quad x = 2t, && y = 4 - t, && z = 3 - 3t \end{aligned}$$

5. [10] Calculate the distance between the point $S = (2, 0, 1)$ and the plane $4x - 2y - 4z = 6$.

6. [10] Sketch the surface bounded below by the elliptical paraboloid $z = x^2 + y^2$ and above by the elliptical paraboloid $z = 2 - x^2 - y^2$. Find the intersection of these two surfaces.

7. [6] What shape is determined by the equation $x^2 - 2x + y^2 + z^2 + 6z - 6 = 0$? Be specific! (That is, tell me where it is, not just what shape it is).