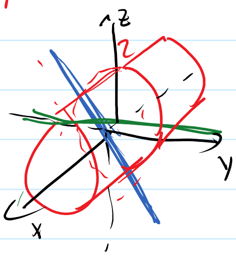
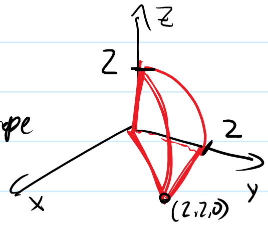
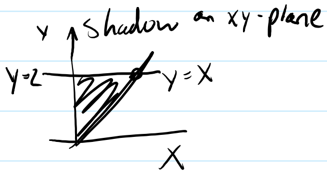
1a) $y^2+z^2=4$, $y=x$, $x=0$:



so the shape is





So, $V=\int_0^2\int_0^y\int_0^{\sqrt{4-y^2}} dz\, dx\, dy$

$=\int_0^2\int_0^y \sqrt{4-y^2}\, dx\, dy$

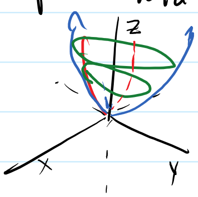
$=\int_0^2 y\sqrt{4-y^2}\, dy$ $\quad u=4-y^2$, $du=-2y\,dy$

$=-\frac{1}{2}\int_4^0 \sqrt{u}\, du$

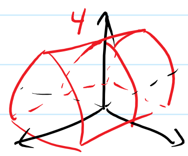
$=-\frac{1}{2}\left(\frac{2}{3}u^{3/2}\Big|_4^0\right)=-\frac{1}{3}\left(0-4^{3/2}\right)=\frac{8}{3}$

b) $z=4x^2+y^2$ & $z=4-3y^2$

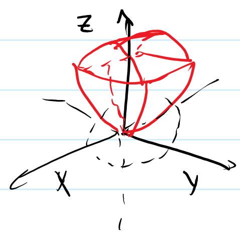
paraboloid



Parabola on zy-plane, Constant in x-direction



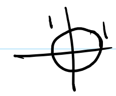
shape is like



intersection is

$4x^2+y^2=4-3y^2$

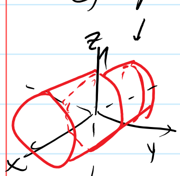
$4x^2+4y^2=4 \rightarrow x^2+y^2=1$

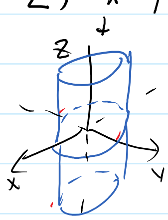


unit circle!

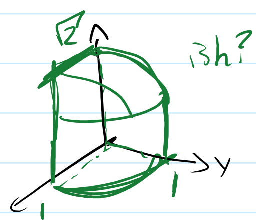
$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{4x^2+y^2}^{4-3y^2} f(x,y,z)\, dz\, dy\, dx$$

c) $y^2+z^2=2$, $x^2+y^2=1$, 1^{st} octant.





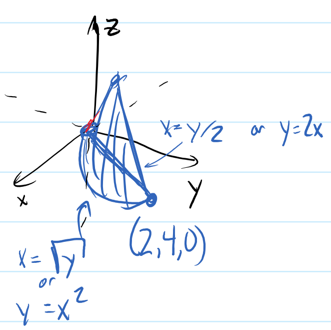
region is



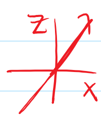
$$\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{2-y^2}} f\, dz\, dy\, dx.$$

2a) $\int_0^4\int_{y/2}^{\sqrt{y}}\int_0^{2x} f\, dz\, dx\, dy$

$z=0$ is xy-plane

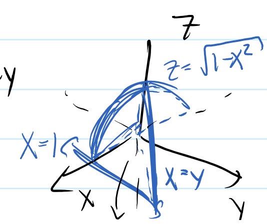


$=\int_0^4\int_{z/2}^{2}\int_{x^2}^{2x} f\, dy\, dx\, dz$



$z=2x$ is a plane making this line on the xz-plane

b) $\int_0^1\int_y^1\int_0^{\sqrt{1-x^2}} f\, dz\, dx\, dy$

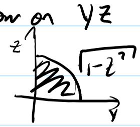


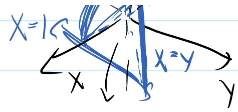
$=\int_0^1\int_0^{\sqrt{1-z^2}}\int_y^{\sqrt{1-z^2}} f\, dx\, dy\, dz$

$z=\sqrt{1-x^2}$ is a half-cylinder

shadow on yz is

$z=\sqrt{1-x^2}$ is a half-cylinder
shadow on yz is

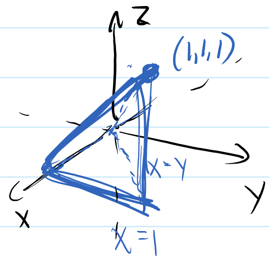




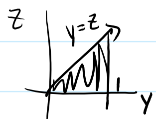
this cheese wedge

c) $\int_0^1\int_y^1\int_0^y f\,dz\,dx\,dy = \int_0^1\int_z^1\int_y^1 f\,dx\,dy\,dz$

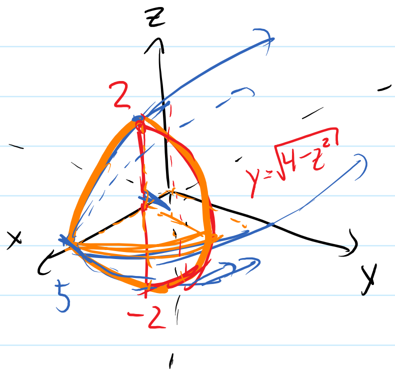
buncha planes



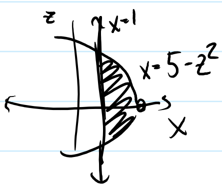
Shadow on y-z is



d) $\int_{-2}^2\int_0^{\sqrt{4-z^2}}\int_1^{5-y^2-z^2} f\,dx\,dy\,dz = \int_{-2}^{2}\int_1^{5-z^2}\int_0^{\sqrt{5-x-z^2}} f\,dy\,dx\,dz$



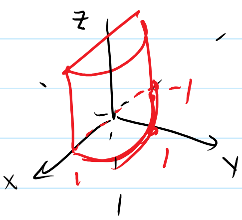
$x=5-y^2-z^2$ is a back-opening paraboloid.
$y=1$ is a plane.
xz-shadow is



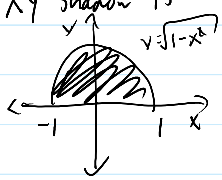
3 a) $\int_{-1}^1\int_0^{\sqrt{1-x^2}}\int_0^4 (x^2+y^2)^{3/2}\,dz\,dy\,dx = \int_0^\pi\int_0^1\int_0^4 (r^2)^{3/2}\, r\,dz\,dr\,d\theta$

$= \int_0^\pi\int_0^1\int_0^4 r^4\,dz\,dr\,d\theta = \int_0^\pi\int_0^1 4r^4\,dr\,d\theta$

$= \int_0^\pi \frac{4}{5}r^5\Big|_0^1 d\theta = \int_0^\pi \frac{4}{5}\,d\theta = 4\frac{\pi}{5}$

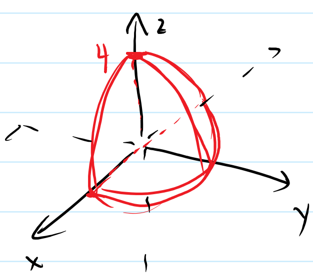


XY-shadow is

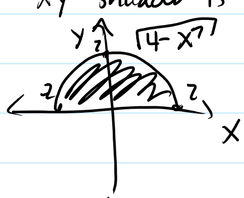


b) $\int_{-2}^2\int_0^{\sqrt{4-x^2}}\int_0^{4-x^2-y^2} y^2\,dz\,dy\,dx$

$4-x^2-y^2 = 4-r^2$ in cylindrical coordinates
$y^2=(r\sin\theta)^2 = r^2\sin^2\theta$



xy-shadow is



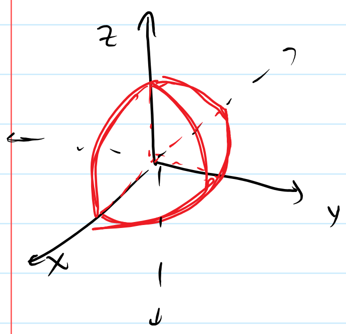
so, integral is
$\int_0^\pi\int_0^2\int_0^{4-r^2} r^2\sin^2(\theta)\,r\,dz\,dr\,d\theta$

c) $\int_{-1}^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} z(x^2+y^2+z^2)^{3/2}\,dz\,dy\,dx$

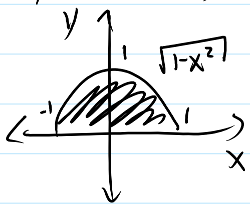
$z(x^2+y^2+z^2)^{3/2}$ in cylindrical coords is

c) $\int_{-1}^{1}\int_{0}\int_{0} z\left(x^2+y^2+z^2\right)^{3/2} dz\,dy\,dx$

$z\left(x^2+y^2+z^2\right)^{3/2}$ in cylindrical coords is $z\left(r^2+z^2\right)^{3/2}$



xy-shadow is



so, integral is

$$\int_0^{\pi}\int_0^{1}\int_0^{\sqrt{1-r^2}} z\left(r^2+z^2\right)^{3/2} r\, dz\, dr\, d\theta$$