Math 241 Homework 9: §15.5, 15.7

1. Use triple integrals to set up an integral of $f(x, y, z)$ over the indicated region. You don't have to integrate, but you might want to anyway.

   (a) The solid in the first octant bounded by the cylinder $y^2 + z^2 = 4$ and the planes $y = x$ and $x = 0$. For practice, try integrating $f(x, y, z) = 1$ over this region.

   (b) The solid enclosed by $z = 4x^2 + y^2$ and $z = 4 - 3y^2$.

   (c) The solid in the first octant inside the cylinders $y^2 + z^2 = 2$ and $x^2 + y^2 = 1$.

2. Change the order of integration in the following integrals to match the new, indicated order. Again, there's no need to integrate unless you want to.

   (a) $\displaystyle\int_0^4 \int_{y/2}^{\sqrt{y}} \int_0^{2x} f(x, y, z)\, dz\, dx\, dy$ to the new order $dy\, dx\, dz$.

   (b) $\displaystyle\int_0^1 \int_y^1 \int_0^{\sqrt{1-x^2}} f(x, y, z)\, dz\, dx\, dy$ to the new order $dx\, dy\, dz$.

   (c) $\displaystyle\int_0^1 \int_y^1 \int_0^y f(x, y, z)\, dz\, dx\, dy$ to the new order $dx\, dy\, dz$.

   (d) $\displaystyle\int_{-2}^2 \int_0^{\sqrt{4-z^2}} \int_1^{5-y^2-z^2} f(x, y, z)\, dx\, dy\, dz$ to the new order $dy\, dx\, dz$.

3. Rewrite the following integrals using cylindrical coordinates. You do not need to integrate.

   (a) $\displaystyle\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \int_0^4 (x^2 + y^2)^{3/2}\, dz\, dy\, dx$. Go ahead and integrate this one for practice.

   (b) $\displaystyle\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} y^2\, dz\, dy\, dx$

   (c) $\displaystyle\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} z(x^2 + y^2 + z^2)^{3/2}\, dz\, dy\, dx$