HW 8 Key

Monday, March 25, 2019 7:25 PM

1) a) $\int_0^1\int_0^2 xy\, dy\, dx = \int_0^1 \frac{xy^2}{2}\Big|_0^2 dx = \int_0^1 2x\, dx = x^2\Big|_0^1 = 1$

b) $\int_0^2\int_0^1 (2x+y)^4\, dx\, dy = \int_0^2\int_0^1 u^4/2\, du\, dy = \int_0^2 u^5/10\Big|_0^1 dy = \int_0^2 (2x+y)^5/10\Big|_0^1 dy = \frac{1}{10}\int_0^2 (2+y)^5 - y^5\, dy$

u-sub: $u = 2x+y$, $du = 2\,dx$

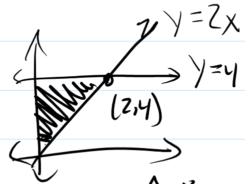
$= \frac{1}{10}\left((2+y)^6/6 - y^6/6\Big|_0^2\right) = \frac{1}{60}\left[(4^6 - 2^6) - (2^6 - 0)\right]$

c) $\int_0^1\int_{x^2}^1 x^3\, dy\, dx = \int_0^1 \left(yx^3\Big|_{x^2}^1\right) dx = \int_0^1 x^3 - x^5\, dx = x^4/4 - x^6/6\Big|_0^1 = 1/4 - 1/6 = 1/12$

d) $\int_0^\pi\int_0^\theta r\, dr\, d\theta = \int_0^\pi r^2/2\Big|_0^\theta d\theta = \int_0^\pi \theta^2/2\, d\theta = \theta^3/6\Big|_0^\pi = \pi^3/6$

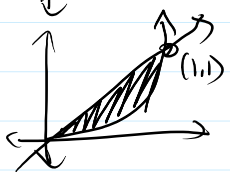
e) $\int_0^\pi\int_0^2 \sin\theta\, dr\, d\theta = \int_0^\pi r\sin\theta\Big|_0^2 d\theta = \int_0^\pi 2\sin\theta\, d\theta = -2\cos\theta\Big|_0^\pi = -2\cos(\pi) + 2\cos(0) = 2+2 = 4$

2) a)



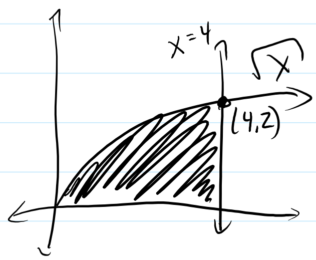
$\int_0^4\int_0^{\frac{1}{2}y} f\, dx\, dy$ or $\int_0^2\int_0^{2x} f\, dy\, dx$

b)



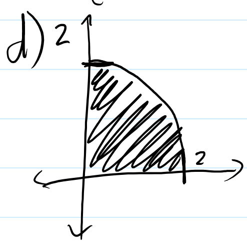
$\int_0^1\int_y^{\sqrt[3]{y}} f\, dx\, dy$ or $\int_0^1\int_{x^3}^{x} f\, dy\, dx$

c)



$\int_0^4\int_0^{\sqrt{x}} f\, dy\, dx$ or $\int_0^2\int_{y^2}^4 f\, dx\, dy$

d)

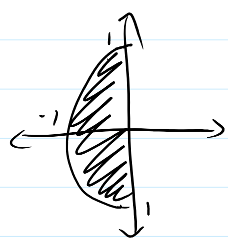


$\int_0^{\frac{\pi}{2}}\int_0^2 f\, r\, dr\, d\theta$

or maybe

$\int_0^2\int_0^{\sqrt{4-x^2}} f\, dy\, dx$

e)

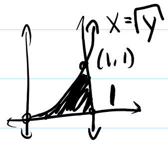


$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\int_0^1 f\, r\, dr\, d\theta$

or

$\int_{-1}^0\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f\, dy\, dx$

3) a) $\int_0^1\int_{\sqrt{y}}^1 \cos(x^3)\, dx\, dy$

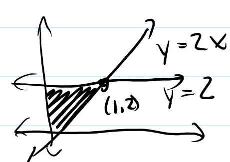


so, $\int_0^1\int_0^{x^2} \cos(x^3)\, dy\, dx = \int_0^1 y\cos(x^3)\Big|_0^{x^2} dx = \int_0^1 x^2\cos(x^3)\, dx$

u-sub $u = x^3$, $du = 3x^2\, dx$

$= \frac{1}{3}\int_0^1 \cos(u)\, du = \frac{1}{3}\sin(x^3)\Big|_0^1 = \frac{1}{3}\sin(1)$

b) $\int_0^1\int_{2x}^2 e^{-y^2}\, dy\, dx$

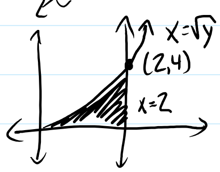


so $\int_0^2\int_0^{\frac{1}{2}y} e^{-y^2}\, dx\, dy = \int_0^2 xe^{-y^2}\Big|_0^{\frac{1}{2}y} dy = \int_0^2 \frac{1}{2}ye^{-y^2}\, dy$

u-sub $u = -y^2$, $du = -2y\, dy$

$= \int_0^2 -\frac{1}{4}e^u\, du = -\frac{1}{4}e^{-y^2}\Big|_0^2 = -\frac{1}{4}(e^{-4} - 1)$

c) $\int_0^4\int_{\sqrt{y}}^2 \frac{3}{x^3+1}\, dx\, dy$



so $\int_0^2\int_0^{x^2} \frac{3}{x^3+1}\, dy\, dx = \int_0^2 \frac{3y}{x^3+1}\Big|_0^{x^2} dx = \int_0^2 \frac{3x^2}{x^3+1}\, dx$

u-sub $u = x^3+1$, $du = 3x^2\, dx$

$= \int_0^2 \frac{1}{u}\, du = \ln(u)\Big|_0^2 = \ln(x^3+1)\Big|_0^2$

$= \ln(9) - \ln(1) = \ln(9)$

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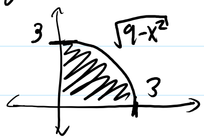
4) a) $\int_0^3 \int_0^{\sqrt{9-x^2}} \sqrt{25-(x^2+y^2)}\, dy\, dx = \int_0^{\frac{\pi}{2}} \int_0^3 \sqrt{25-r^2}\, r\, dr\, d\theta \underset{\substack{u = 25-r^2 \\ du = -2r\,dr}}{=} \int_0^{\frac{\pi}{2}} -\frac{1}{2} \int_0^3 \sqrt{u}\, du\, d\theta$



$= -\frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{2}{3} u^{\frac{3}{2}} \Big|_0^3 d\theta = -\frac{1}{3} \int_0^{\frac{\pi}{2}} (25-r^2)^{\frac{3}{2}} \Big|_0^3 d\theta = -\frac{1}{3} \int_0^{\frac{\pi}{2}} (25-9)^{3/2} - (25)^{3/2}\, d\theta$

$= -\frac{1}{3} \int_0^{\frac{\pi}{2}} (16)^{3/2} - (25)^{3/2}\, d\theta = -\frac{1}{3} \int_0^{\frac{\pi}{2}} 4^3 - 5^3\, d\theta = -\frac{1}{3} \int_0^{\frac{\pi}{2}} 64 - 125\, d\theta$

$= -\frac{1}{3} \cdot \left(-61\, \theta \Big|_0^{\frac{\pi}{2}}\right) = \frac{61\pi}{6}$

b) $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{x^2+y^2}\, dy\, dx = \int_0^{2\pi} \int_0^2 \sqrt{r^2}\, r\, dr\, d\theta = \int_0^{2\pi} \int_0^2 r^2\, dr\, d\theta = \int_0^{2\pi} \frac{r^3}{3} \Big|_0^2 d\theta$

$= \int_0^{2\pi} \frac{8}{3}\, d\theta = \frac{16\pi}{3}$