HW 7 Key

Saturday, March 2, 2019 11:04 PM

1a) $x^2 - 2y^2 - z^2 = 1$ @ $(2,-1,1)$ $\nabla f|_p = \langle 2x, -4y, -2z\rangle|_p = \langle 4, 4, -2\rangle$

plane is $4(x-2) + 4(y-1) - 2(z-1) = 0$

$4x - 8 + 4y + 4 - 2z + 2 = 0$

$4x + 4y - 2z = 2$ or $2x + 2y - z = 1$

b) $x^2 + xy - y^2 - z^2 = 4$ @ $(2,1,1)$ $\nabla f|_p = \langle 2x+y, x-2y, -2z\rangle|_p = \langle 5, 0, -2\rangle$

plane $5(x-2) - 2(z-1) = 0$

$5x - 10 - 2z + 2 = 0$

$5x - 2z = 8$

c) $x^3 - y^2 + z^4 = 1$ @ $(1,1,1)$ $\nabla f|_p = \langle 3x^2, -2y, 4z^3\rangle|_p = \langle 3, -2, 4\rangle$

plane $3(x-1) - 2(y-1) + 4(z-1) = 0$

$3x - 2y + 4z = 5$

d) $2(x-2)^2 + (y-1)^2 + (z-3)^2 = 10$ @ $(3,3,5)$ $\nabla f|_p = \langle 4(x-2), 2(y-1), 2(z-3)\rangle|_p = \langle 4, 4, 4\rangle$

plane $4(x-3) + 4(y-3) + 4(z-5) = 0$

$x + y + z = 11$

e) $x^2 + 3xy - 2y^2 + z^2 = 0$ @ $(1,-1,2)$ $\nabla f|_p = \langle 2x+3y, 3x-4y, 2z\rangle = \langle -1, 7, 4\rangle$

plane $-(x-1) + 7(y+1) + 4(z-2) = 0$

$-x + 7y + 4z = 0$

2a) $f(x,y) = x^3 + 3y^2 - 6xy$ $f_x = 3x^2 - 6y$ $f_y = 6y - 6x$

$3x^2 - 6y = 0$

$-6x + 6y = 0 \Rightarrow x = y \Rightarrow 3x^2 - 6x = 0 \Rightarrow 3x(x-2) = 0$ so $(0,0)$ & $(2,2)$ are crit points

$f_{xx} = 6x$ $f_{yy} = 6$ $f_{xy} = -6$

$(0\ 0)$: $f_{xx}f_{yy} - f_{xy}^2 = 0 - 36 < 0$ so a saddle at $(0,0,0)$

$(2\ 2)$ $f_{xx}f_{yy} - f_{xy}^2 = 12\cdot 6 - 36 > 0$. $f_{xx} > 0$, so a min at $(2,2,-4)$

b) $f(x,y) = 2x^2 + y^2 + 2xy^2$ $f_x = 4x + 2y^2$ $f_y = 2y + 4xy$

$4x + 2y^2 = 0$

$2y + 4xy = 0 \rightarrow 2y(1+2x) = 0$ so either $y = 0 \rightarrow 4x = 0 \rightarrow x = 0$

$x = -\frac{1}{2} \rightarrow -2 + 2y^2 = 0 \rightarrow 2y^2 = 2 \Rightarrow y = \pm 1$

$f_{xx} = 4$ $f_{yy} = 2 + 4x$ $f_{xy} = 4y$

$(0,0)$: $f_{xx}f_{yy} - f_{xy}^2 = 4\cdot 2 - 0 > 0$. $f_{xx} > 0$, so min at $(2,2,28)$

$(-\frac{1}{2}, -1)$: " $= 4\cdot 0 - 16 < 0$ saddle at $(-\frac{1}{2}, -1, \frac{1}{2})$

$(-\frac{1}{2}, 1)$: $= 4\cdot 0 - 16 < 0$ saddle at $(-\frac{1}{2}, 1, \frac{1}{2})$

c) $f(x,y) = 2x^2y - 8xy + y^2 + 5$ $f_x = 4xy - 8y$ $f_y = 2x^2 - 8x + 2y$

$4xy - 8y = 0 \Rightarrow 4y(x-2) = 0$ so either $y = 0 \rightarrow 2x^2 - 8x = 0 \rightarrow 2x(x-4) = 0 \rightarrow x = 0$, $x = 4$

$2x^2 - 8x + 2y = 0$ or $x = 2 \rightarrow 8 - 16 + 2y = 0 \rightarrow 2y = 8 \rightarrow y = 4$

$f_{xx} = 4y$ $f_{yy} = 2$ $f_{xy} = 4x - 8$

$(0,0)$ $f_{xx}f_{yy} - f_{xy}^2 = 0\cdot 2 - (4\cdot 0 - 8)^2 < 0$ saddle at $(0,0,5)$

$(4,0)$ " $= 0\cdot 2 - (16-8)^2 < 0$ saddle at $(4,0,5)$

$(2,4)$ $= 16\cdot 2 - (8-8)^2 > 0$ $f_{xx} > 0$ min at $(2,4,-11)$

3a) $f(x,y) = 18x^2 - 6x + 3 - 24xy + 16y^2$ $f_x = 36x - 6 - 24y$ $f_y = -24x + 32y$

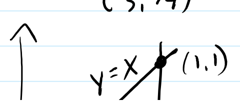
$36x - 6 - 24y = 0$

$-24x + 32y = 0$ $32y = 24x \Rightarrow y = \frac{3}{4}x \Rightarrow 36x - 6 - 24(\frac{3}{4}x) = 0 \rightarrow 6x - 1 - 4(\frac{3}{4}x) = 0 \rightarrow x = \frac{1}{3}$

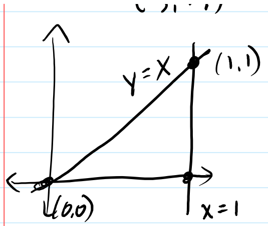
so $y = \frac{1}{4}$

$f_{xx} = 36$ $f_{yy} = 32$ $f_{xy} = -24$

$(\frac{1}{3}, \frac{1}{4})$ $f_{xx}f_{yy} - f_{xy}^2 = 36\cdot 32 - (24)^2 > 0$. $f_{xx} > 0$ so this is a local min.



along $y = 0$, $f(x,0) = 18x^2 - 6x + 3 - 24(x)(0) + 16(0)^2$



along $y=0$, $f(x,0) = 18x^2 - 6x + 3 - 24(x)(0) + 16(0)^2$
$f'(x,0) = 36x - 6$ crit. point at $x = 1/6$.
along $x=1$, $f(1,y) = 18 - 6 + 3 - 24y + 16y^2$
$f'(1,y) = -24 + 32y$ c.p. at $y = 1/4$
along $y=x$, $f(x,x) = 18x^2 - 6x + 3 - 24x^2 + 16x^2 = 10x^2 - 6x + 3$
$f'(x,x) = 20x - 6$ c.p. at $x = 3/10$

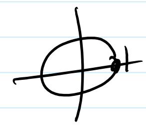
So we have to check the following points:

Interior point $(1/3, 1/4)$. $f(1/3, 1/4) = 2$
Corner points $(0,0)$ $f(0,0) = 3$
$(1,0)$ $f(1,0) = 15$
$(1,1)$ $f(1,1) = 7$

Boundary c.p.'s: $(1/6, 0)$ $f(1/6, 0) = 5/2$
$(1, 1/4)$ $f(1, 1/4) = 10$
$(3/10, 3/10)$ $f(3/10, 3/10) = 21/10$

So, absolute min is at $(1/3, 1/4, 2)$. Abs. max at $(1, 0, 15)$.

3b) $f(x,y) = 3x^2 + 6y^2 - 2x$ $f_x = 6x - 2$ $f_y = 12y$



$6x - 2 = 0$, $12y = 0$ $\Rightarrow$ $x = 1/3$, $y = 0$ is the only interior c.p.

The boundary is $x^2 + y^2 = 1$; that is, $y = \sqrt{1-x^2}$ and $y = -\sqrt{1-x^2}$.

$f(x, \sqrt{1-x^2}) = 3x^2 + 6(\sqrt{1-x^2})^2 - 2x = 3x^2 + 6(1-x^2) - 2x = 3x^2 + 6 - 6x^2 - 2x = -3x^2 - 2x + 6$

$f'(x, \sqrt{1-x^2}) = -6x - 2$ so cp at $x = -1/3$, $y = \sqrt{1-(1/3)^2} = \sqrt{8/9}$ or $\frac{2\sqrt{2}}{3}$

$f(x, -\sqrt{1-x^2})$ is the same function, so we get a cp at $x = -1/3$, $y = -\frac{2\sqrt{2}}{3}$.

So, we check:

$f(1/3, 0) = 3(\frac{1}{3})^2 - 2(\frac{1}{3}) = -1/3$

$f(-\frac{1}{3}, \sqrt{8/9}) = 3(-\frac{1}{3})^2 + 6(\sqrt{8/9})^2 - 2(-\frac{1}{3}) = 3(\frac{1}{9}) + 6(8/9) + 2/3 = \frac{1}{3} + 16/3 + 2/3 = 19/3$

$f(-\frac{1}{3}, -\sqrt{8/9}) = 3(-\frac{1}{3})^2 + 6(-\sqrt{8/9})^2 - 2(-\frac{1}{3}) = 19/3$ also

Also we need to check the semicircle's endpoints.

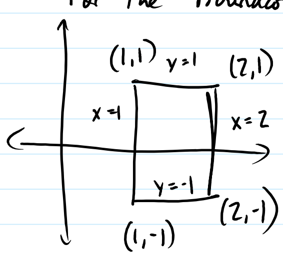
$f(1,0) = 3 + 0 - 2 = 1$
$f(-1,0) = 3 + 0 + 2 = 5$

So abs. min is at $(1/3, 0, -1/3)$ and abs. maxes are at $\left(-\frac{1}{3}, \pm\frac{2\sqrt{2}}{3}, \frac{19}{3}\right)$

3c) $f(x,y) = 2x^3 + 5x^2 + 4xy^2$ $f_x = 6x^2 + 10x + 4y^2$ $f_y = 8xy$

$6x^2 + 10x + 4y^2 = 0$
$8xy = 0$ $\rightarrow$ $x = 0 \rightarrow 4y^2 = 0 = y = 0$
or
$y = 0 \rightarrow 6x^2 + 10x = 0 \rightarrow 2x(3x+5) = 0 \rightarrow x = 0$ or $\rightarrow x = -5/3$

For the boundary,



along $x=1$, $f(1,y) = 2 + 5 + 4y^2$
$f'(1,y) = 8y$ so c.p. is @ $y = 0$
along $x=2$, $f(2,y) = 16 + 20 + 8y^2$
$f'(2,y) = 16y^2$ so c.p. @ $y = 0$
along $y=1$, $f(x,1) = 2x^3 + 5x^2 + 4x$
$f'(x,1) = 6x^2 + 10x + 4 = 2(3x^2 + 5x + 2) = 2(3x+2)(x+1)$ so $x = -1$ (out of bounds) $x = -2/3$ (also out of bounds)
along $y=-1$, $f(x,-1) = 2x^3 + 5x^2 + 4x$
so same c.p.'s, still out of bounds.

All told, check

Interior points: $f(0,0) = 0$ $f(-5/3, 0) = 2(-5/3)^3 + 5(-5/3)^2 + 0 = 125/27$ (≈ 4.6)

Corners : $f(1,1)=11$ $f(1,-1)=11$ $f(2,1)=16+20+4=40$ $f(2,-1)=40$
boundary : $f(1,0)=7$ $f(2,0)=36$

So abs. min is $(0,0,0)$ and abs. maxes are $(2,1,40)$ and $(2,-1,40)$