HW 6 Key

Tuesday, February 26, 2019 2:01 PM

1a) $\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^4+y^4}$ along $x=0$, $\lim_{(0,y)\to(0,0)} \frac{0}{y^4}=0$. Along $y=x$, $\lim_{(x,x)\to(0,0)} \frac{x^4}{2x^4}=\frac{1}{2}$. Limit doesn't exist!

b) $\lim_{(x,y)\to(1,1)} \frac{x^2-2xy+y^2}{x-y} = \lim_{(x,y)\to(1,1)} \frac{(x-y)^2}{(x-y)} = \lim_{(x,y)\to(1,1)} (x-y) = 0$

c) $\lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}}$ along $x=0$, $\lim_{(0,y)\to(0,0)} \frac{0}{\sqrt{y^2}}=0$. Along $y=0$, $\lim_{(x,0)\to(0,0)} \frac{x}{\sqrt{x^2}}=1$. DNE!

2a) $\lim_{(x,y)\to(0,0)} \frac{x^3-3x^2+x^2y-3y^2}{x^2+y^2}$ along $x=0$, $\lim_{(0,y)\to(0,0)} \frac{0-0+0-3y^2}{0+y^2}=-3$ So if the limit exists as stated, it must be -3.

b) $\lim_{(x,y)\to(0,0)} \frac{1-(x^2+y^2-1)^2}{4-(x^2+y^2+2)^2}$ along $x=0$, $\lim_{(0,y)\to(0,0)} \frac{1-(y^2-1)^2}{4-(y^2+2)^2} = \lim_{y\to0} \frac{1-(y^4-2y^2+1)}{4-(y^4+4y^2+4)} = \lim_{y\to0} \frac{-y^4+2y^2}{-y^4-4y^2} = \lim_{y\to0} \frac{-\cancel{y^2}(y^2+2)}{-\cancel{y^2}(y^2-4)} = -\frac{1}{2}$

c) $\lim_{(x,y)\to(0,0)} \frac{x^3+\sin(x^2+y^2)}{x^2+y^2}$ along $x=0$, $\lim_{(0,y)\to(0,0)} \frac{\sin(y^2)}{y^2} \overset{\to\frac{0}{0}}{\underset{\text{L'Hôpital}}{=}} \lim_{y\to0} \frac{\cancel{2y}\cos(y^2)}{\cancel{2y}} = 1$

3a) $f(x,y)=x^3y+y^2+2xy$. $f_x=3x^2y+2y$ $f_y=x^3+2y+2x$ $f_{xy}=3x^2+2$

b) $f(x,y)=ye^y+\sin(x+y)$ $f_x=\cos(x+y)$ $f_y=ye^y+e^y+\cos(x+y)$ $f_{xy}=-\sin(x+y)$

c) $f(x,y)=x\ln(y)-\frac{x^2}{x+y}$ $f_x=\ln(y)-\frac{(x+y)2x-x^2}{(x+y)^2}$ $f_y=\frac{x}{y}-\frac{(x+y)(0)-x^2(1)}{(x+y)^2}=\frac{x}{y}+\frac{x^2}{(x+y)^2}$

$f_{xy}=\underset{\text{(easier)}}{f_{yx}}=\frac{1}{y}+\frac{(x+y)^2\cdot2x-x^2\cdot2(x+y)}{(x+y)^4}$

d) $f(x,y)=2^{xy}+x^2y^3$ $f_x=y\cdot\ln(2)\cdot2^{xy}+2xy^3$ $f_y=x\cdot\ln(2)\cdot2^{xy}+3y^2x^2$

$f_{xy}=y\cdot\ln(2)\cdot x\ln(2)2^{xy}+\ln(2)2^{xy}+6xy^2=xy\ln(2)^2 2^{xy}+\ln(2)2^{xy}+6xy^2$

4) $f(x,y)=x^2+y^3$

a) $\nabla f=\langle 2x,3y^2\rangle$. So $\nabla f(2,1)=\langle 4,3\rangle$

b) $\frac{\vec v}{|\vec v|}=\frac{\vec v}{\sqrt{1+4}}=\langle\frac{1}{\sqrt5},\frac{-2}{\sqrt5}\rangle$. $\nabla f(1,1)=\langle 2,3\rangle$. $D_{\vec u}f=\nabla f\cdot\vec u=\frac{2}{\sqrt5}-\frac{6}{\sqrt5}=-\frac{4}{\sqrt5}$

c) $\nabla f(-1,2)=\langle -2,12\rangle$, this is the direction of maximal increase. Derivative here is $|\nabla f(-1,2)|=\sqrt{148}$

5) $f(x,y)=xe^y+y^2$

$\nabla f=\langle f_x,f_y\rangle=\langle e^y,xe^y+2y\rangle$. $\nabla f(-1,0)=\langle e^0,-e^0+2(0)\rangle=\langle 1,-1\rangle$.

$\frac{\vec v}{|\vec v|}=\frac{\vec v}{\sqrt{9+16}}=\langle\frac{3}{5},-\frac{4}{5}\rangle$. So $D_{\vec u}f=\nabla f\cdot\vec u=\frac{3}{5}+\frac{4}{5}=\frac{7}{5}$.

6) $f(x,y)=y\sin(xy)$. So $\nabla f=\langle y^2\cos(xy),\ \sin(xy)+yx\cos(xy)\rangle$.

Max'l decrease at $(0,1)$ is $-\nabla f(0,1)=-\langle 1^2\cos(0),\ \sin(0)+1\cdot0\cos(0)\rangle=\langle -1,0\rangle$

$D_{\vec u}f$ at that point is $-|\nabla f|=-1$.

7) $z=x^3y+y^2x-x$ where $x=e^{st}$ $y=te^{st}$

$\frac{\partial z}{\partial x}=3x^2y+y^2-1$ $\frac{\partial z}{\partial y}=x^3+2yx$ $\frac{\partial x}{\partial s}=te^{st}$ $\frac{\partial y}{\partial s}=ste^{st}$

so $\frac{\partial z}{\partial s}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial s}=(3x^2y+y^2-1)(te^{st})+(x^3+2yx)(ste^{st})$