Math 241 Homework 6: §14.1-14.5

1. Calculate the following limits, or prove that they don't exist.

   (a) $\lim_{(x,y)\to(0,0)} \dfrac{x^3y}{x^4+y^4}$

   (b) $\lim_{(x,y)\to(1,1)} \dfrac{x^2-2xy+y^2}{x-y}$

   (c) $\lim_{(x,y)\to(0,0)} \dfrac{x}{\sqrt{x^2+y^2}}$

2. Assume that the following limits exist (they do!). Find out what they must be.

   (a) $\lim_{(x,y)\to(0,0)} \dfrac{x^3-3x^2+x^2y-3y^2}{x^2+y^2}$

   (b) $\lim_{(x,y)\to(0,0)} \dfrac{1-(x^2+y^2-1)^2}{4-(x^2+y^2+2)^2}$

   (c) $\lim_{(x,y)\to(0,0)} \dfrac{x^3+\sin(x^2+y^2)}{x^2+y^2}$

3. Calculate the partial derivatives f_x, f_y, and f_{xy} for the following functions.

   (a) $f(x,y) = x^3y + y^2 + 2xy$

   (b) $f(x,y) = ye^y + \sin(x+y)$

   (c) $f(x,y) = x\ln(y) - \frac{x^2}{x+y}$

   (d) $f(x,y) = 2^{xy} + x^2y^3$

4. Let $f(x,y) = x^2 + y^3$.

   (a) Calculate $\nabla f(2,1)$

   (b) Calculate the directional derivative of $f(x,y)$ at $P = (1,1)$ in the direction of $\mathbf{v} = \langle 1, -2\rangle$.

   (c) At the point $P = (-1,2)$, in which direction is f increasing the most? What is the directional derivative in that direction?

5. Calculate the directional derivative of $f(x,y) = xe^y + y^2$ at the point $(-1,0)$ in the direction of the vector $\mathbf{v} = \langle 3, -4\rangle$.

6. In which direction is the function $f(x,y) = y\sin(xy)$ decreasing the most at the point $(0,1)$? What is the directional derivative in that direction?

7. Let $z = x^3y + y^2x - x$ where $x = e^{st}$ and $y = te^{st}$. Calculate $\partial z/\partial s$ in terms of s and t.