HW 5 Key

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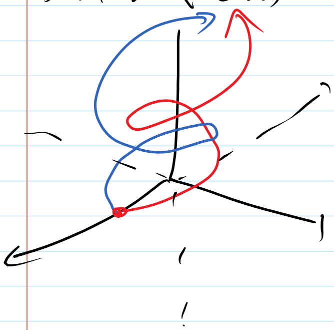
1) $\vec{r}(t) = \langle 2\cos t, 4\sin t, t^2 \rangle$. x & y make an ellipse with radii 2 & 4. z increases in a parabolic fashion



if $t=0$, get $\langle 2,0,0 \rangle$

climbing in both directions.

2) $\begin{aligned} t^2 &= s^2 - s \\ t &= 2s-2 \\ t^2+t &= s^2+s-2 \end{aligned} \Rightarrow (2s-2)^2 = s^2 - s \Rightarrow 4s^2 - 8s + 4 = s^2 - s$

$3s^2 - 7s + 4 = 0$

solutions are $s = \frac{7 \pm \sqrt{49-48}}{6} = \frac{7 \pm 1}{6} = 1, \frac{4}{3}$

if $s=1$, $t=0$ $\Rightarrow (0,0,0)$ if $s = 4/3$, $t = 2/3 \Rightarrow \left(4/9, 2/3, 10/9\right)$

3) $\begin{aligned} t^2 - 2t &= t \\ t^3 &= t^2 + 6t \\ 2t+1 &= t+4 \rightarrow t = 3 \end{aligned}$

$\vec{r}_1(3) = \langle 3, 27, 7 \rangle$

$\vec{r}_2(3) = \langle 3, 27, 7 \rangle$

so the particles collide after 3s

4) $\lim_{t\to 0} \left\langle \sin t, e^t, \frac{\sin(t)}{4t} \right\rangle = \left\langle \lim_{t\to 0} \sin t, \lim_{t\to 0} e^t, \lim_{t \to 0} \frac{\sin(t)}{4t} \right\rangle = \left\langle 0, 1, \lim_{t\to 0} \frac{\cos(t)}{4} \right\rangle = \langle 0, 1, 1/4 \rangle$ (L'Hôpital)

5) a) $\vec{r}'(t) = \langle -6\sin(2t), -8\sin(2t), 10\cos(2t) \rangle$.

$L = \int_0^2 \sqrt{36\sin^2(2t) + 64\sin^2(2t) + 100\cos^2(2t)}\, dt = \int_0^2 \sqrt{100(\sin^2(2t) + \cos^2(2t))}\, dt = \int_0^2 10\, dt = 10t\Big|_0^2 = 20$

for tangent, $\vec{r}'(1) = \langle -6\sin(2), -8\sin(2), 10\cos(2) \rangle$. $\vec{r}(1) = \langle 3\cos(2), 4\cos(2), 5\sin(2) \rangle$

so $\begin{aligned} x &= 3\cos 2 - (6\sin 2)t \\ y &= 4\cos 2 - (8\sin 2)t \\ z &= 5\sin 2 + (10\cos 2)t \end{aligned}$ nasty but it works.

b) $\vec{r}'(t) = \langle 2t+1, 2t+1, 2t+1 \rangle$

$L = \int_0^1 \sqrt{(2t+1)^2 + (2t+1)^2 + (2t+1)^2}\, dt = \int_0^1 \sqrt{3(2t+1)^2}\, dt = \int_0^1 \sqrt{3}(2t+1)\, dt = \sqrt{3}\left(t^2 + t\Big|_0^1\right) = 2\sqrt{3}$

for tangent, $\vec{r}'(0) = \langle 1, 1, 1 \rangle$. $\vec{r}(0) = \langle 1, -2, 0 \rangle$

so $\begin{aligned} x &= 1 + t \\ y &= -2 + t \\ z &= t \end{aligned}$

c) $\vec{r}'(t) = \langle e^t + 1, e^t - 1, 2e^{\frac{t}{2}} \rangle$

$L = \int_0^1 \sqrt{(e^t+1)^2 + (e^t-1)^2 + (2e^{\frac{t}{2}})^2}\, dt = \int_0^1 \sqrt{e^{2t} + 2e^t + 1 + e^{2t} - 2e^t + 1 + 4e^t}\, dt = \int_0^1 \sqrt{2e^{2t} + 4e^t + 2}\, dt$

$= \int_0^1 \sqrt{2(e^t+1)^2}\, dt = \sqrt{2} \int_0^1 e^t + 1\, dt = \sqrt{2}\left(e^t + t\Big|_0^1\right)$

$= \sqrt{2}\left[(e+1) - (1+0)\right] = e\sqrt{2}$.

for tangent, $\vec{r}'(1) = \langle e+1, e-1, 2e^{\frac{1}{2}} \rangle$. $\vec{r}(1) = \langle e+1, e-1, 4e^{\frac{1}{2}} \rangle$

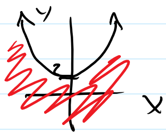
so $x = e+1 + t(e+1)$
$y = e-1 + t(e-1)$
$z = 4e^{\frac{1}{2}} + t(2e^{\frac{1}{2}})$

6) a) $\vec{v}(t) = \langle 3t^2, e^t, \cos(t) \rangle$ so $\vec{v}(2) = \langle 12, e^2, \cos(2) \rangle$

b) $s(2) = |\vec{v}(2)| = \sqrt{144 + e^4 + \cos^2(2)}$ whatever that is.

c) $\vec{a}(t) = \langle 6t, e^t, -\sin(t) \rangle$.
so $\vec{a}(-1) = \langle -6, \frac{1}{e}, -\sin(-1) \rangle$

7) a) domain is $x^2 - y + 2 \geq 0 \iff x^2 + 2 \geq y$. So this:



with a hard border

range is $[0, \infty)$

b) domain is $y \neq 0$. so everything but the x-axis.
range is $\mathbb{R}$.

c) range is $\mathbb{R}$, for domain, we need $(x-1)^2 + (y+2)^2 + z^2 - 9 > 0$
so $(x-1)^2 + (y+2)^2 + z^2 > 9$

in other words, everything outside of the sphere of radius 3 centered at $(1, -2, 0)$