Math 241 Homework 5: Ch. 13, §14.1

1. Sketch and describe the vector function $\mathbf{r}(t) = \langle 2\cos(t), 4\sin(t), t^2\rangle$.

2. Find all the points of intersection of the curves traced by

$$\mathbf{r}_1(t) = \langle t^2, t, t^2 + t\rangle \qquad \mathbf{r}_2(t) = \langle t^2 - t, 2t - 2, t^2 + t - 2\rangle$$

3. Two particles travel along the curves described by

$$\mathbf{r}_1(t) = \langle t^2 - 2t, t^3, 2t + 1\rangle \qquad \mathbf{r}_2(t) = \langle t, t^2 + 6t, t + 4\rangle$$

   Do the particles collide? When?

4. Let $\mathbf{r}(t) = \left\langle \sin(t), e^t, \dfrac{\sin(t)}{4t}\right\rangle$. Calculate $\lim_{t\to 0} \mathbf{r}(t)$.

5. Find the arc length of the following functions over the given intervals, and then find a parameterisation for the tangent line at the given point.

   (a) $\mathbf{r}(t) = \langle 3\cos(2t), 4\cos(2t), 5\sin(2t)\rangle$. Find arc length from $t = 0$ to $t = 2$, and the tangent line at $t = 1$.

   (b) $\mathbf{r}(t) = \langle t^2 + t + 1, t^2 + t - 2, t^2 + t\rangle$. Find arc length along $[0, 1]$, and tangent line at $t = 0$.

   (c) $\mathbf{r}(t) = \langle e^t + t, e^t - t, 4e^{t/2}\rangle$. Find arc length along $0 \le t \le 1$, and tangent line at $t = 1$.

6. Suppose $\mathbf{r}(t) = \langle t^3, e^t, \sin(t)\rangle$ gives the position of a particle at a time t.

   (a) What is the velocity of the particle after 2 seconds?

   (b) What is the speed of the particle after 2 seconds (no need to simplify)?

   (c) What was the acceleration of the particle 1 second ago?

7. Find the domains and ranges of the following multivariable functions. Sketch the domain, when possible.

   (a) $f(x, y) = \sqrt{x^2 - y + 2}$.

   (b) $f(x, y) = \frac{x}{y}$.

   (c) $f(x, y, z) = \dfrac{z}{\sqrt{(x-1)^2 + (y+2)^2 + z^2 - 9}}$