HW 4 Key

Saturday, February 2, 2019 5:22 PM

1)a) $x_1 = 2+s$, $y_1 = 3-s$, $z_1 = 1+2s$; $x_2 = 2t$, $y_2 = 1+3t$, $z_2 = t$ / $z_1 = z_2 \Rightarrow t = 1+2s$

$x_1 = x_2 \Rightarrow 2+s = 2+4s$

$0 = 3s$, so $s=0$ and $t = 1+2(0) = 1$.

Using $s=0$ & $t=1$,

$P_1 = (2,3,1)$ $P_2 = (2,4,0)$, so these lines don't intersect. They're not parallel as their vectors $\vec{v}_1$ & $\vec{v}_2$ don't agree on direction. So they're $\boxed{\text{skew}}$

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = \vec{i}(-1-6) - \vec{j}(1-4) + \vec{k}(3+2) = \langle -7, 3, 5 \rangle. \text{ So,}$$

$$D = \left| \frac{\overrightarrow{P_1P_2} \cdot (\vec{v}_1 \times \vec{v}_2)}{|\vec{v}_1 \times \vec{v}_2|} \right| = \left| \frac{\langle 0,1,-1\rangle \cdot \langle -7,3,5\rangle}{\sqrt{49+9+25}} \right| = \left| \frac{0+3-5}{\sqrt{83}} \right| = \frac{2}{\sqrt{83}}$$

b) $x_1 = 2+2s$, $y_1 = 1-3s$, $z_1 = 4-s$; $x_2 = 1-4t$, $y_2 = 1+6t$, $z_2 = 2t$

vectors are $\vec{v}_1 = \langle 2,-3,-1\rangle$, $\vec{v}_2 = \langle -4,6,2\rangle$. $\vec{v}_2 = -2\vec{v}_1$, so l_1 & l_2 are $\boxed{\text{parallel}}$

to find distance, pick a point on l_1, $P_1 = (2,1,4)$ will do. Then, our formula for distance to $l_2 = (1,1,0) + \langle -4,6,2\rangle t$

$$D = \left| \frac{\overrightarrow{P_1P_2} \times \vec{v}_2}{\vec{v}_2} \right| = \left| \frac{\langle -1,0,-4\rangle \times \langle -4,6,2\rangle}{\sqrt{16+36+4}} \right|$$

sidebar: $\langle -1,0,-4\rangle \times \langle -4,6,2\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & -4 \\ -4 & 6 & 2 \end{vmatrix} = \vec{i}(0+24) - \vec{j}(-1-16) + \vec{k}(-6+0) = \langle 24, 17, -6\rangle$

$$= \left| \frac{\langle 24, 17, -6\rangle}{\sqrt{56}} \right| = \frac{\sqrt{24^2+17^2+36}}{\sqrt{56}},$$ whatever the hell that is.

c) $x_1 = 4+2t$, $y_1 = -t$, $z_1 = 4+3t$; $x_2 = -4+s$, $y_2 = 11-4s$, $z_2 = -15+5s$

$x_1 = x_2 \Rightarrow 4+2t = -4+s$, $8+2t = s$. Then, $y_1 = y_2 \Rightarrow -t = 11-4(8+2t)$

$-t = 11-32-8t$

$7t = -21$

$t = -3$ so $s = 2$.

Using $t=-3$ $s=2$, we get

$P_1 = (-2,3,-5)$ $P_2 = (-2,3,-5)$ Same point! They $\boxed{\text{intersect}}$ here.

2) $l_1 = \begin{cases} x = 2+t \\ y = 0 \\ z = -1+t \end{cases}$ $l_2 = \begin{cases} x = 4 \\ y = 2s \\ z = 1+s \end{cases}$ setting y's equal makes $s=0$, setting x's equal makes $t=2$. So, intersection is $(4,0,1)$.

3) $S = (2,-1,0)$

a) $x-y+2z=4$, use $\left|\frac{\overrightarrow{PS} \cdot \vec{n}}{|\vec{n}|}\right|$ with $P = (0,0,2)$ (or any other pt on the plane) and $\vec{n} = \langle 1,-1,2\rangle$.

$$D = \left| \frac{\langle 2,-1,-2\rangle \cdot \langle 1,-1,2\rangle}{|\langle 1,-1,2\rangle|} \right| = \left| \frac{2+1-4}{\sqrt{1+1+4}} \right| = \frac{1}{\sqrt{6}}$$

b) $x=t$, $y=2-t$, $z=1+t$ use $\frac{|\overrightarrow{PS} \times \vec{v}|}{|\vec{v}|}$ where $P = (0,2,1)$ (or equivalent). $\overrightarrow{PS} = (2,-3,-1)$ so and $\vec{v} = \langle 1,-1,1\rangle$

$$\overrightarrow{PS} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & -1 \\ 1 & -1 & 1 \end{vmatrix} = \vec{i}(-3-1) - \vec{j}(2+1) + \vec{k}(-2+3) = \langle -4, -3, 1\rangle$$

$$D = \frac{|\langle -4,-3,1\rangle|}{|\langle 1,-1,1\rangle|} = \frac{\sqrt{16+9+1}}{\sqrt{3}} = \frac{\sqrt{26}}{\sqrt{3}}$$

c) $3x+2y-z=-2$. use $\left|\frac{\overrightarrow{PS} \cdot \vec{n}}{|\vec{n}|}\right|$ where $P = (0,0,2)$, $\vec{n} = \langle 3,2,-1\rangle$ $\overrightarrow{PS} = \langle 2,-1,-2\rangle$

$$D = \left| \frac{\langle 2,-1,-2\rangle \cdot \langle 3,2,-1\rangle}{\langle 3,2,-1\rangle} \right| = \left| \frac{6-2+2}{\sqrt{9+4+1}} \right| = \frac{6}{\sqrt{14}}$$

4) $\overrightarrow{PS} = -\overrightarrow{SP}$. In the line formula, $\left|\frac{\overrightarrow{PS} \times \vec{v}}{|\vec{v}|}\right| = \left|\frac{-(\overrightarrow{PS} \times \vec{v})}{|\vec{v}|}\right| = \frac{|(-\overrightarrow{PS}) \times \vec{v}|}{|\vec{v}|} = \frac{|\overrightarrow{SP} \times \vec{v}|}{|\vec{v}|}$. thanks to those lovely absolute values.

In the plane formula, $\left|\frac{\overrightarrow{PS} \cdot \vec{n}}{|\vec{n}|}\right| = \left|\frac{-(\overrightarrow{PS} \cdot \vec{n})}{|\vec{n}|}\right| = \left|\frac{(-\overrightarrow{PS}) \cdot \vec{n}}{|\vec{n}|}\right| = \left|\frac{\overrightarrow{SP} \cdot \vec{n}}{|\vec{n}|}\right|$

absolute values,

In the plane formula, $\left|\frac{\vec{PS}\cdot\vec{n}}{|\vec{n}|}\right| = \left|-\frac{(\vec{PS}\cdot\vec{n})}{|\vec{n}|}\right| = \left|\frac{(-\vec{PS})\cdot\vec{n}}{|\vec{n}|}\right| = \left|\frac{\vec{SP}\cdot\vec{n}}{|\vec{n}|}\right|$

5) $l_1 = \begin{cases} x = 3+t \\ y = 2t \\ z = -1-t \end{cases}$ $\quad l_2 = \begin{cases} x = 3t \\ y = 1-t \\ z = -2+t \end{cases}$ their vectors are $\vec{v}_1 = \langle 1, 2, -1\rangle$ and $\vec{v}_2 = \langle 3, -1, 1\rangle$.
$\vec{v}_1 \cdot \vec{v}_2 = 3-2-1 = 0$, so they're orthogonal, thus l_1 & l_2 are perpendicular.

To find their intersection, solve
$$\begin{aligned} 3+t &= 3s \\ 2t &= 1-s \Rightarrow s = 1-2t \\ -1-t &= -2+s \Rightarrow -1-t = -2+1-2t \Rightarrow t=0,\ s=1 \end{aligned}$$
so intersection is $(3, 0, -1)$

6) a) $-2x+y-z=4$ & $-2x+y-z=1$. I'll use $P=(0,0,-4)$, $S=(0,0,-1)$, $\vec{n}=\langle -2,1,-1\rangle$

$$D = \left|\frac{\vec{PS}\cdot\vec{n}}{|\vec{n}|}\right| = \left|\frac{\langle 0,0,3\rangle\cdot\langle -2,1,-1\rangle}{\sqrt{4+1+1}}\right| = \frac{3}{\sqrt{6}}$$

b) $3x-y+2z=-1$ $\quad 6x-2y+4z=5$. I'll use $P=(0,1,0)$ $S=(0,0,\frac{5}{4})$, $\vec{n}=\langle 3,-1,2\rangle$

$$D = \left|\frac{\vec{PS}\cdot\vec{n}}{|\vec{n}|}\right| = \left|\frac{\langle 0,-1,\frac{5}{4}\rangle\cdot\langle 3,-1,2\rangle}{\sqrt{9+1+4}}\right| = \left|\frac{0+1+\frac{5}{2}}{\sqrt{14}}\right| = \frac{\frac{7}{2}}{\sqrt{14}} = \frac{7}{2\sqrt{14}}$$

c) $Ax+By+Cz=D$ $\quad Ax+By+Cz=E$. I'll use $P=(0,0,\frac{D}{C})$ $S=(0,0,\frac{E}{C})$, $\vec{n}=\langle A,B,C\rangle$

$$D = \left|\frac{\vec{PS}\cdot\vec{n}}{|\vec{n}|}\right| = \left|\frac{\langle 0,0,\frac{E}{C}-\frac{D}{C}\rangle\cdot\langle A,B,C\rangle}{\sqrt{A^2+B^2+C^2}}\right| = \left|\frac{E-D}{\sqrt{A^2+B^2+C^2}}\right|,$$ or $\frac{|E-D|}{|\vec{n}|}$ if you like.

7) Hemisphere of radius 3.

$z=0 \cap z=\sqrt{9-x^2-y^2}$
$\Downarrow$
$0 = \sqrt{9-x^2-y^2}$
$\Rightarrow 9 = x^2+y^2$, which is the circle of radius 3 on the xy-plane.