Math 241 Homework 4: §12.5 & 12.6

1. Determine whether the following lines are parallel or skew lines. If they are, find the shortest distance between them. If they are not, find the point of intersection. Don't worry about simplifying awful radicals or fractions.

   (a) l_1 given by $x_1 = 2 + s$, $y_1 = 3 - s$, $z_1 = 1 + 2s$, and l_2 given by $x_2 = 2t$, $y_2 = 1 + 3t$, $z_2 = t$.

   (b) l_1 given by $x_1 = 2 + 2s$, $y_1 = 1 - 3s$, $z_1 = 4 - s$, and l_2 given by $x_2 = 1 - 4t$, $y_2 = 1 + 6t$, $z_2 = 2t$.

   (c) l_1 given by $x_1 = 4 + 2t$, $y_1 = -t$, $z_1 = 4 + 3t$, and l_2 given by $x_2 = -4 + s$, $y_2 = 11 - 4s$, $z_2 = -15 + 5s$.

2. Calculate the distance between the following lines.

$$l_1 = \begin{cases} x &= 2 + t \\ y &= 0 \\ z &= -1 + t \end{cases}, \qquad l_2 = \begin{cases} x &= 4 \\ y &= 2t \\ z &= 1 + t \end{cases}$$

3. Calculate the distance between the point $S = (2, -1, 0)$ and the following objects.

   (a) The plane $x - y + 2z = 4$.

   (b) The line given by $x = t$, $y = 2 - t$, $z = 1 + t$.

   (c) The plane $3x + 2y - z = -2$.

4. The formula giving distance from a point to a line and the formula giving distance from a point to a plane both make use of a vector $\overrightarrow{PS}$ where S is the point of interest and P is a point on the line or plane. Explain in your own words why the vector $\overrightarrow{SP}$ would also work fine in both of these formulae, and produce the same distance.

5. Prove that the two lines

$$l_1 = \begin{cases} x &= 3 + t \\ y &= 2t \\ z &= -1 - t \end{cases}, \qquad l_2 = \begin{cases} x &= 3t \\ y &= 1 - t \\ z &= -2 + t \end{cases}$$

   are perpendicular. Find their intersection.

6. Calculate the distance between the following pairs of parallel planes.

   (a) $-2x + y - z = 4$ and $-2x + y - z = 1$

   (b) $3x - y + 2z = -1$ and $6x - 2y + 4z = 5$

   (c) $Ax + By + Cz = D$ and $Ax + By + Cz = E$. Doing this correctly should provide you with a formula for this sort of problem.

7. Sketch the solid bounded below by the xy-plane and above by the hemisphere $z = \sqrt{9 - x^2 - y^2}$. Find the intersection of the plane and the hemisphere.

8. For more practice on quadric surfaces, try out exercise 1-12 on pg. 732 of Thomas.