Math 241 Homework 3: §12.4, 12.5

1. Suppose $\mathbf{u}$ and $\mathbf{v}$ are non-parallel vectors. Let $\mathbf{w} = \mathbf{u} \times \mathbf{v}$ be their cross product. What is the value of $\mathbf{u} \cdot \mathbf{w}$? What about $\mathbf{v} \cdot \mathbf{w}$?

2. Let $P = (1, 0, -1)$, $Q = (5, -2, -1)$, and $R = (3, 0, -2)$.

   (a) Find the vector $\overrightarrow{PQ}$.

   (b) Calculate $cos\theta$ where θ is the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

   (c) There are two unit vectors that are each orthogonal to both $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. What are they?

   (d) What is the area of triangle $\triangle PQR$?

3. Let $P = (1, 4, 1)$, $Q = (2, 3, 5)$, and $R = (1, 3, 6)$.

   (a) What is the distance from point Q to the z-axis?

   (b) Find the vector $\overrightarrow{QP}$.

   (c) Calculate the value of $\angle PQR$.

   (d) What is the area of $\triangle PQR$?

4. Let $\mathbf{u} = \langle 7, -2, -4 \rangle$ and $\mathbf{v} = \langle 5, 1, 0 \rangle$. Find two unit vectors that are orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

5. What is the area of the parallelogram three of whose vertices are $P = (2, 1, 5)$, $Q = (-3, 2, 5)$ and $R = (-3, 1, 6)$?

6. Calculate the volume of the parallelepiped which has the vectors $\langle 2, 3, 1 \rangle$, $\langle 0, 1, -1 \rangle$ and $\langle 0, 0, -3 \rangle$ as adjacent edges.

7. Find parametric equations for the lines that fit the following descriptions.

   (a) The line through the points $P = (0, 3, -2)$ and $Q = (2, -9, 6)$.

   (b) The line parallel to the line given by $x = -1 + 3t$, $y = 4 - t$, $z = 1 - t$ passing through the point $(-2, 1, 3)$.

   (c) Give **three different parameterisations** for the line parallel to the line given by $x = 1 + 2t$, $y = 4 - t$, and $z = 6 + 2t$ passing through the point $(-2, 0, 5)$.

8. Find the point where the lines below intersect.

$$l_1 : \begin{cases} x = & 3t \\ y = & t \\ z = & -1 + t \end{cases} \qquad l_2 : \begin{cases} x = & t \\ y = & 2 - t \\ z = & -2 + t \end{cases}$$

9. Suppose I gave you two parameterisations for lines and told you that they intersect, but you thought I was full of it. How could you prove me wrong?

10. Find equations for the planes that fit the following descriptions.

    (a) The plane through $(0, 2, -1)$ normal to $\mathbf{n} = \langle 3, -2, -1 \rangle$.

(b) The plane through $(1, -1, 3)$ parallel to the plane $3x + y + z = 7$.

(c) The plane through $(2, 4, 5)$, $(1, 5, 7)$, and $(-1, 6, 8)$.

11. Find the three intersections given by the above problem; that is, the intersection between your answers for (a) and (b), for (b) and (c), and for (a) and (c).

12. Calculate the angles between the same pairs of planes as for the previous problem.

13. Let $S = (0, 3, -1)$. Calculate its distance from

(a) The line parameterised by $x = t$, $y = 3 - 2t$, $z = 1 + t$.

(b) The plane $3x - 2y + 4z = 1$.