HW 2 Key

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1a) $\vec{u} = \langle 1,4 \rangle$, $\vec{v} = \langle -2,6 \rangle$

$\vec{u}\cdot\vec{v} = -2+24 = 22.$ $\theta = \cos^{-1}\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\right) = \cos^{-1}\left(\frac{22}{\sqrt{17}\sqrt{40}}\right) = \cos^{-1}\left(\frac{11}{\sqrt{170}}\right)$

$\text{proj}_{\vec{v}}\vec{u} = \left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\right)\vec{v} = \left(\frac{22}{40}\right)\vec{v} = \left\langle -\frac{11}{10}, \frac{66}{20} \right\rangle$

$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 4 & 0 \\ -2 & 6 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} 4 & 0 \\ 6 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 0 \\ -2 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 4 \\ -2 & 6 \end{vmatrix} = 0\vec{i} - 0\vec{j} + \vec{k}(6+8) = \langle 0,0,14 \rangle$

1b) $\vec{u} = \langle 2,-3 \rangle$ $\vec{v} = \langle 0,2 \rangle$

$\vec{u}\cdot\vec{v} = 0-6 = -6$ $\theta = \cos^{-1}\left(\frac{-6}{\sqrt{13}\cdot 2}\right) = \cos^{-1}\left(\frac{-3}{\sqrt{13}}\right)$

$\text{proj}_{\vec{v}}\vec{u} = \left(\frac{-6}{4}\right)\vec{v} = \langle 0,-3 \rangle$

$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 0 \\ 0 & 2 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} -3 & 0 \\ 2 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & -3 \\ 0 & 2 \end{vmatrix} = 0\vec{i} - 0\vec{j} + \vec{k}(4-0) = \langle 0,0,4 \rangle$

1c) $\vec{u} = \langle \frac{1}{2}, 1 \rangle$ $\vec{v} = \langle -3,2 \rangle$

$\vec{u}\cdot\vec{v} = -\frac{3}{2} + 2 = \frac{1}{2}$ $\theta = \cos^{-1}\left(\frac{\frac{1}{2}}{\sqrt{\frac{5}{4}}\sqrt{13}}\right) = \cos^{-1}\left(\frac{\sqrt{5}}{4\sqrt{13}}\right)$

$\text{proj}_{\vec{v}}\vec{u} = \left(\frac{\frac{1}{2}}{13}\right)\vec{v} = \left\langle -\frac{3}{26}, \frac{1}{13} \right\rangle$

$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2} & 1 & 0 \\ -3 & 2 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} \frac{1}{2} & 0 \\ -3 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} \frac{1}{2} & 1 \\ -3 & 2 \end{vmatrix} = \langle 0,0,4 \rangle$

1d) $\vec{u} = \langle -1,2,4 \rangle$ $\vec{v} = \langle 3,-2,1 \rangle$

$\vec{u}\cdot\vec{v} = -3-4+4 = -3$ $\theta = \cos^{-1}\left(\frac{-3}{\sqrt{21}\sqrt{14}}\right)$

$\text{proj}_{\vec{v}}\vec{u} = \left(\frac{-3}{14}\right)\vec{v} = \left\langle -\frac{9}{14}, \frac{3}{7}, \frac{-3}{14} \right\rangle$

$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 4 \\ 3 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 4 \\ -2 & 1 \end{vmatrix}\vec{i} - \begin{vmatrix} -1 & 4 \\ 3 & 1 \end{vmatrix}\vec{j} + \begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix}\vec{k} = (2+8)\vec{i} - (-1-12)\vec{j} + (2-6)\vec{k}$

$= \langle 10, 13, -4 \rangle$

1e) $\vec{u} = \langle 2,2,0 \rangle$ $\vec{v} = \langle -1,4,-1 \rangle$

$\vec{u}\cdot\vec{v} = -2+8+0 = 6$ $\theta = \cos^{-1}\left(\frac{6}{\sqrt{8}\sqrt{18}}\right) = \cos^{-1}\left(\frac{6}{2\sqrt{2}\,3\sqrt{2}}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\text{proj}_{\vec{v}}\vec{u} = \left(\frac{6}{18}\right)\vec{v} = \left\langle -\frac{1}{3}, \frac{4}{3}, \frac{-1}{3} \right\rangle$

$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & k \\ 2 & 2 & 0 \\ -1 & 4 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 4 & -1 \end{vmatrix}\vec{i} - \begin{vmatrix} 2 & 0 \\ -1 & -1 \end{vmatrix}\vec{j} + \begin{vmatrix} 2 & 2 \\ -1 & 4 \end{vmatrix}\vec{k} = (-2-0)\vec{i} - (-2-0)\vec{j} + (8+2)\vec{k} = \langle -2, 2, 10\rangle$

2) any two vectors whose dot product is 0.

3) $\langle -3, 1\rangle$ and $\langle -6, 2\rangle$ work; any vectors who are scalar multiples of each other.

4) $A(3,-3,-2)$, $B(2,-1,1)$ $C(4,2,2)$. $\vec{BA} = \langle 1,-2,-3\rangle$. $\vec{BC} = \langle 2,3,1\rangle$

$\theta = \cos^{-1}\left(\frac{\vec{BA}\cdot\vec{BC}}{|\vec{BA}||\vec{BC}|}\right) = \cos^{-1}\left(\frac{2-6-3}{\sqrt{14}\sqrt{14}}\right) = \cos^{-1}\left(\frac{-7}{14}\right) = \cos^{-1}\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$

5) a) false; it's a scalar. b) true

c) false: try $\vec{u} = \langle 1,1\rangle$, $\vec{v} = \langle 2,2\rangle$, $\vec{w} = \langle 3,1\rangle$

d) false: try $\vec{u} = \langle 1,-1\rangle$, $\vec{v} = \langle 1,1\rangle$

6) $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$, so $\frac{2\pi}{3} = \frac{\vec{u}\cdot\vec{v}}{3\cdot 4}$, so $\vec{u}\cdot\vec{v} = 8\pi$

7) a) makes no sense, scalar × scalar

b) fine

c) fine

d) same as (a), can't be done

e) fine