Math 241 Homework 2: §12.3, some 12.4

1. For each of the following pairs of vectors, calculate the following

   - The dot product $\mathbf{u} \cdot \mathbf{v}$
   - The smallest angle between the two vectors (you may give your answer in terms of inverse trig functions)
   - The projection $\text{proj}_{\mathbf{v}} \mathbf{u}$
   - The cross product $\mathbf{u} \times \mathbf{v}$

   (a) $\mathbf{u} = \langle 1, 4 \rangle$, $\mathbf{v} = \langle -2, 6 \rangle$
   (b) $\mathbf{u} = \langle 2, -3 \rangle$, $\mathbf{v} = \langle 0, 2 \rangle$
   (c) $\mathbf{u} = \langle 1/2, 1 \rangle$, $\mathbf{v} = \langle -3, 2 \rangle$
   (d) $\mathbf{u} = \langle -1, 2, 4 \rangle$, $\mathbf{v} = \langle 3, -2, 1 \rangle$
   (e) $\mathbf{u} = \langle 2, 2, 0 \rangle$, $\mathbf{v} = \langle -1, 4, -1 \rangle$

2. Give an example of two orthogonal vectors. What makes them orthogonal?

3. Give an example of two nonequal parallel vectors. What makes them parallel?

4. Let $A = (3, -3, -2)$, $B = (2, -1, 1)$ and $C = (4, 2, 2)$. What is the value of the angle $\angle ABC$?

5. Determine whether the following statements are true or false. If false, give the reason, or a counterexample.

   (a) The dot product of two vectors is a vector.
   (b) If θ is the smallest angle between $\mathbf{u}$ and $\mathbf{v}$, then $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos\theta$.
   (c) If $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$ and $\mathbf{u} \neq \mathbf{0}$, then $\mathbf{v} = \mathbf{w}$.
   (d) If $\mathbf{u} \cdot \mathbf{v} = \mathbf{0}$ and $\mathbf{u} \neq \mathbf{0}$, then $\mathbf{v} = \mathbf{0}$.

6. Find $\mathbf{u} \cdot \mathbf{v}$ if $|\mathbf{u}| = 3$, $|\mathbf{v}| = 4$, and the angle between $\mathbf{u}$ and $\mathbf{v}$ is $2\pi/3$ radians.

7. Determine which of the following expressions make sense; if they don't, say why they don't make sense.

   (a) $(\mathbf{u} \cdot \mathbf{v}) \times (\mathbf{a} \cdot \mathbf{b})$
   (b) $(\mathbf{u} \times \mathbf{v}) \cdot (\mathbf{a} \times \mathbf{b})$
   (c) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$
   (d) $\mathbf{u} \times (\mathbf{v} \cdot \mathbf{w})$
   (e) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$