HW 11 Key

Tuesday, April 23, 2019   3:01 PM

1a) $\int_C xy^2\,ds$ over $\overline{(0,0),(1,2)}$ $\vec r(t)=\langle t,2t\rangle$, $0\le t\le 1$. $|\vec v(t)|=|\langle 1,2\rangle|=\sqrt5$

$= \int_0^1 (t)(2t)^2(\sqrt5)\,dt = \sqrt5\int_0^1 4t^3\,dt = \sqrt5\, t^4\Big|_0^1 = \sqrt5$

b) $\int_C \sqrt{x}\,y\,ds$ over $\overline{(0,2)\,(1,-1)}$. $\vec r(t)=\langle t, -3t+2\rangle$ $0\le t\le 1$. $|\vec v(t)|=|\langle 1,-3\rangle|=\sqrt{10}$

$= \int_0^1 \sqrt t(-3t+2)\sqrt{10}\,dt = \sqrt{10}\int_0^1 -3t^{3/2}+2t^{1/2}\,dt = \sqrt{10}\left(-3\cdot\frac25 t^{5/2}+2\cdot\frac23 t^{3/2}\Big|_0^1\right) = \sqrt{10}\left(-\frac65+\frac43\right) = \sqrt{10}\left(-\frac{18}{15}+\frac{20}{15}\right)$
$= \frac{2\sqrt{10}}{15}$

c) $\int_C \sqrt{x^2+y^2}\,ds$ over 2. $\vec r(t)=\langle 2\cos t, 2\sin t\rangle$, $0\le t\le\pi$. $|\vec v(t)|=|\langle -2\sin t, 2\cos t\rangle| = \sqrt{4\sin^2t+4\cos^2t}=2$

$= \int_0^\pi \sqrt{4\cos^2t+4\sin^2t}\cdot 2\,dt = \int_0^\pi 4\,dt = 4\pi$

d) $\int_C \sqrt{xy+y+z}\,ds$ over $\overline{(0,0,1),(1,1,-2)}$. $\vec r(t)=\langle t, t, 1-3t\rangle$ $0\le t\le 1$. $|\vec v(t)|=|\langle 1,1,-3\rangle|=\sqrt{11}$

$= \int_0^1 \sqrt{(t)(t)+t+(1-3t)}\sqrt{11}\,dt = \sqrt{11}\int_0^1\sqrt{t^2-2t+1}\,dt = \sqrt{11}\int_0^1\sqrt{(t-1)^2}\,dt = \sqrt{11}\int_0^1 t-1\,dt = \sqrt{11}\left(\frac{t^2}{2}-t\Big|_0^1\right) = -\frac{\sqrt{11}}{2}$

e) $\int_C (x^2\cos(y)+y)\,dx + (x^2\cos(y)-y)\,dy$ over $\overline{(1,0),(0,1)}$. $\vec r(t)=\langle 1-t, t\rangle$, $0\le t\le 1$. $x'(t)=-1$. $y'(t)=1$.

$= \int_0^1 ((1-t)^2\cos(t)+t)(-1) + ((1-t)^2\cos(t)-t)(1)\,dt$

$= \int_0^1 -(1-t)^2\cos t - t + (1-t)^2\cos t - t\,dt = \int_0^1 -2t\,dt = -t^2\Big|_0^1 = -1$

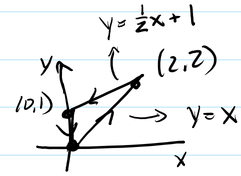
f) $\int_C (x-2y)\,dx + (2x+y)\,dy$ over 1. $\vec r(t)=\langle\cos t,\sin t\rangle$, $0\le t\le\pi$. $x'(t)=-\sin t$, $y'(t)=\cos t$

$= \int_0^\pi (\cos t-2\sin t)(-\sin t) + (2\cos t+\sin t)\cos t\,dt$

$= \int_0^\pi -\sin t\cos t + 2\sin^2t + 2\cos^2t + \sin t\cos t\,dt$

$= \int_0^\pi 2\sin^2t + 2\cos^2t\,dt = \int_0^\pi 2\,dt = 2\pi$

2a) $\oint_C (y^2+x^3-2x)\,dx + (2xy+x^2-3\cos(y^2+1))\,dy$ over

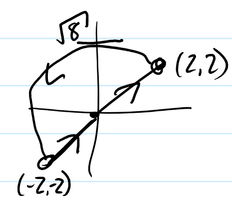


$\partial P/\partial y = 2y$ $\partial Q/\partial x = 2y+2x$

$\oint_C P\,dx+Q\,dy = \iint_R (2y+2x)-(2y)\,dA = \int_0^2\int_x^{\frac12x+1} 2x\,dy\,dx = \int_0^2 2xy\Big|_x^{\frac12x+1}dx = \int_0^2 2x\left(\frac12x+1-x\right)dx$

$= \int_0^2 x^2+2x-2x^2\,dx = \int_0^2 -x^2+2x\,dx = -\frac{x^3}{3}+x^2\Big|_0^2 = -\frac83+4 = 4/3$

b) $\oint_C (y+\sin(x))\,dx + (3x-y^3\cos y)\,dy$ over

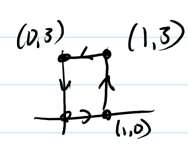


$\partial P/\partial y = 1$ $\partial Q/\partial x = 3$

$\oint_C P\,dx+Q\,dy = \iint_R 3-1\,dA$

$= \int_{\pi/4}^{5\pi/4}\int_0^{\sqrt8} 2\,r\,dr\,d\theta = \int_{\pi/4}^{5\pi/4} r^2\Big|_0^{\sqrt8} d\theta = \int_{\pi/4}^{5\pi/4} 8\,d\theta = 8\left(\frac{5\pi}{4}-\frac{\pi}{4}\right) = 8\pi$

c) $\oint_C (\cos(x)+\sin(y)-xy^3)\,dx + (x\cos(y)-x^2y^2+e^{y^2+1})\,dy$ over



$\partial P/\partial y = \cos(y)-3xy^2$ $\partial Q/\partial x = \cos(y)-2xy^2$

$\oint_C P\,dx+Q\,dy = \iint_R (\cos(y)-2xy^2)-(\cos y-3xy^2)\,dA = \int_0^3\int_0^1 xy^2\,dx\,dy = \int_0^3 y^2\frac{x^2}{2}\Big|_0^1 dy$

$= \int_0^3 \frac{y^2}{2}\,dy = \frac{y^3}{6}\Big|_0^3 = \frac92$