HW 10 Key

Wednesday, April 10, 2019 11:22 AM

1) a) $(\rho,\varphi,\theta) = (6, \frac{\pi}{6}, \frac{\pi}{6})$ $\quad (r,\theta,z) = (\rho\sin\varphi, \theta, \rho\cos\varphi) = (6\sin(\frac{\pi}{6}), \frac{\pi}{6}, 6\cos(\frac{\pi}{6})) = (3, \frac{\pi}{6}, 3\sqrt{3})$

$(x,y,z) = (\rho\sin\varphi\cos\theta, \rho\sin\varphi\sin\theta, \rho\cos\varphi) = (6\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}, 6\cdot\frac{1}{2}\cdot\frac{1}{2}, 3\sqrt{3}) = (\frac{3\sqrt{3}}{2}, \frac{3}{2}, 3\sqrt{3})$

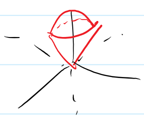
b) $(\rho,\varphi,\theta) = (4, \frac{\pi}{6}, \frac{7\pi}{6})$ $\quad (r,\theta,z) = (4\sin(\frac{\pi}{6}), \frac{7\pi}{6}, 4\cos(\frac{\pi}{6})) = (2, \frac{7\pi}{6}, 2\sqrt{3})$

$(x,y,z) = (4\sin(\frac{\pi}{6})\cos(\frac{7\pi}{6}), 4\sin(\frac{\pi}{6})\sin(\frac{7\pi}{6}), 2\sqrt{3}) = (4\cdot\frac{1}{2}\cdot\frac{-\sqrt{3}}{2}, 4\cdot\frac{1}{2}\cdot\frac{-1}{2}, 2\sqrt{3}) = (-\sqrt{3}, -1, 2\sqrt{3})$

c) $(\rho,\varphi,\theta) = (4, \frac{\pi}{6}, \frac{\pi}{2})$ $\quad (r,\theta,z) = (4\sin\frac{\pi}{6}, \frac{\pi}{2}, 4\cos\frac{\pi}{6}) = (2, \frac{\pi}{2}, 2\sqrt{3})$

$(x,y,z) = (4\sin\frac{\pi}{6}\cos\frac{\pi}{2}, 4\sin\frac{\pi}{6}\sin\frac{\pi}{2}, 2\sqrt{3}) = (0, 2, 2\sqrt{3})$

2a) $\displaystyle\int_0^{2\pi}\int_0^{\pi/6}\int_0^3 \rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta$

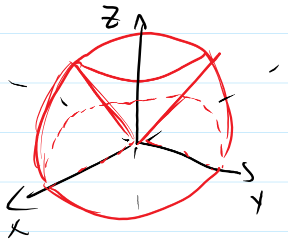


b) inside $x^2+y^2+z^2=4$, which is $\rho=2$.

$z=\sqrt{x^2+y^2}$ in spherical becomes $\rho\cos\varphi = \sqrt{\rho^2\sin^2\varphi\cos^2\theta + \rho^2\sin^2\varphi\sin^2\theta}$

$\rho\cos\varphi = \sqrt{\rho^2\sin^2\varphi(\cos^2\theta+\sin^2\theta)}$

$\rho\cos\varphi = \rho\sin\varphi \rightarrow \cos\varphi = \sin\varphi \rightarrow \varphi = \frac{\pi}{4}$



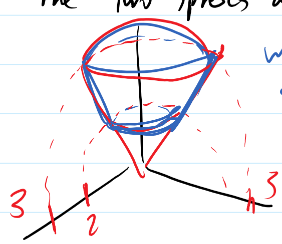
we're after the top hemisphere with the cone cut out.

$$\int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^2 \rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta$$

c) $z=\sqrt{(x^2+y^2)/3}$ becomes $\rho\cos\varphi = \sqrt{(\rho^2\sin^2\varphi\cos^2\theta + \rho^2\sin^2\varphi\sin^2\theta)/3} = \frac{1}{\sqrt{3}}\rho\sin\varphi$ as before,

so $\frac{\cos\varphi}{\sin\varphi} = \frac{1}{\sqrt{3}} \rightarrow \cot\varphi = \frac{1}{\sqrt{3}} \rightarrow \varphi = \frac{\pi}{3}$.

the two spheres are $\rho=2$ and $\rho=3$. So,



we want this slice of the sno-cone.

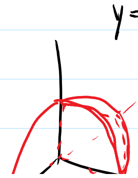
$$\int_0^{2\pi}\int_0^{\pi/3}\int_2^3 \rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta$$

3a) $\displaystyle\int_{-1}^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} z(x^2+y^2+z^2)^{3/2}\, dz\, dy\, dx$

inside: $z(x^2+y^2+z^2)^{3/2}$ becomes $\rho\cos\varphi(\rho^2)^{3/2} = \rho^4\cos\varphi$.

bounds: $z=0$ is xy-plane, $z=\sqrt{1-x^2-y^2}$ is the top hemisphere of radius 1.

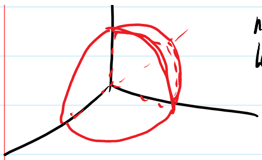
$y=0$, $y=\sqrt{1-x^2}$ and $x=-1, x=1$ form the top semicircle of radius 1 on the xy-plane.



need this lemon wedge.

$$\int_0^{\pi}\int_0^{\pi/2}\int_0^1 \rho^4\cos\varphi\ \rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta$$

$\int^{\pi}\int^{\pi/2}\int^{1}$



need this lemon wedge.

$$\int_0 \int_0 \int_0 \rho^4 \cos\varphi \; \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

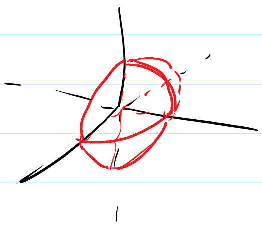
$$= \int_0^{\pi} \int_0^{\frac{\pi}{2}} \int_0^{1} \rho^6 \cos\varphi \sin\varphi \, d\rho \, d\varphi \, d\theta.$$

b) $\displaystyle\int_0^3 \int_0^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} x^2 \, dz \, dy \, dx$

inside: x^2 becomes $\rho^2 \sin^2\varphi \cos^2\theta$.

bounds: $z = -\sqrt{9-x^2-y^2}$ and $z = \sqrt{9-x^2-y^2}$ describe the sphere $\rho = 3$

$y=0$, $y = \sqrt{9-x^2}$, $x=0$, $x=3$ describe the top semicircle of radius 3 in the xy-plane



like (a) but under the xy-plane too. Half a sphere.

$$\int_0^{\pi} \int_0^{\pi} \int_0^{3} \rho^2 \sin^2\varphi \cos^2\theta \; \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$= \int_0^{\pi} \int_0^{\pi} \int_0^{3} \rho^4 \sin^3\varphi \cos^2\theta \, d\rho \, d\varphi \, d\theta$$