Name:

Test 3: Ch. 7

Complete the following problems to the best of your ability. **SHOW ALL OF YOUR WORK.** Unshown work will not be graded. You may use a calculator.

1. [45] We are, once again, working with a standard 52-card deck (13 denominations, 4 suits).

   (a) If we are dealt five cards, what is the probability that we'll be dealt a full house (three of one denomination, two of another)?

   (b) Let's say we draw ten cards as an experiment. The ten cards we picked were

   $$A\spadesuit\ 10\heartsuit\ 4\spadesuit\ K\diamondsuit\ K\clubsuit\ J\clubsuit\ 5\spadesuit\ J\heartsuit\ Q\spadesuit\ 2\clubsuit$$

   Write a table that gives a relative frequency distribution for drawing a given suit in this experiments.

   (c) We take the ten cards that we just drew and shuffle them, and then draw one. Let C be the event that I drew a club, and T be the event that I drew a two.

      i. List the elements of the set C.

      ii. What does $P(T|C)$ mean in english? Evaluate it.

2. Marty McFly's DeLorean is on the fritz[1]. The Flux Capacitor is randomly hopping the car between the future (1985) and the past (1955) every hour. If the car is in the past, there is a 45% chance that it will go back to the future. If it's in the future, there's a 80% chance that it will go back to the past.

   (a) [10] Let State One be that the car is in the past, and State Two be that the car is in the future. Draw a picture for this Markov System, and write its Transition Matrix.

   (b) [15] Right now, the DeLorean is in the future. Use the transition matrix to find the probability that the car is in the future 5 hours from now (you can use your calculator, but write what you plugged in for full credit).

   (c) [15] What is the steady-state vector v_∞ for this system?

[1] Credit to grad student Rob Vandermolen for the premise of this problem

3. [20] A 14th-century village priest has developed a test for witchcraft, but it's not flawless and he wants to make sure he's not chasing the wrong people out of town (they don't burn them in this village, they're quite progressive).

   The word from the King is that rougly 4% of citizens are witches or warlocks. The priest's test catches a witch 98% of the time, but it has a 30% false positive rate. If the test returns the result that someone is a witch, what is the likelihood that they actually are?

4. [Extra Credit, 10] How many people need to be in a room so that at least three people are mutual acquaintances, or three people mutually do not know each other? Give a rigorous proof of your answer.