Name: MATH 242 §101

Final Exam

Complete the following problems to the best of your ability. **SHOW ALL OF YOUR WORK.** Unshown work will not be graded.

1. Find solutions to the following differential equations. If initial conditions are given, find an explicit solution.

   (a) [5] $xy^2y' = x^3 + y^3$; $y(1) = 3$

   (b) [5] $3y^2y' + y^3 = e^{-x}$; $y(0) = 2$

   (c) [5] $y'' - 5y' - 6y = 0$; $y(0) = 1$, $y'(0) = -2$

(d) [4] $y^{(4)} + 4y'' = 0$

(e) [6] $y'' + 3y' + 2y = 2e^{-x}$

(f) [6] $x'' + x = \sin(2t)$; $x(0) = x'(0) = 0$.

2. [14] A 1000L tank contains 100kg of salt dissolved in water. 2% salt water is pumped into the tank at 5 L/s, and the stirred mixture is pumped out at the same rate. How long will it take until only 40kg of salt remains in the tank?

3. [14] A 3kg mass is attached to the end of a spring. A force of 15N will move the mass by 20cm (0.2m). If the mass is set in motion with an initial position of 2m to the right and an initial velocity of 3m/s to the left, what is the mass's position after 4 seconds?

4. [10] Find all equilibrium solutions of the equation $y' = 2y^3 + 6y^2 - 8y$ and determine whether they are stable or unstable.

5. [10] Given the initial value problem $dy/dx = 2x + 3y - xy$; $y(0) = 0$, use Euler's improved method with a step size of $h = 1$ to estimate $y(2)$.

6. Suppose that the population $P(t)$ of a country satisfies the differential equation $dP/dt = kP(100 - P)$ where k is a constant. Its population in 2000 was 40 million, and was growing at a rate of 1 million per year.

   (a) [10] Predict this country's population for the year 2020.

   (b) [3] At this rate, how long will the population take to reach 70 million?

   (c) [3] What is the carrying capacity of this population?

7. [14] Find a general solution for the initial value problem $tx'' + (t-2)x' + x = 0$; $x(0) = 0$.