Name:

Math 111 001/S01, Spring 2017
Final Exam

Complete the following problems to the best of your ability. **SHOW ALL OF YOUR WORK.** Unshown work will not be graded. You may use a calculator.

1. [15] Simplify the following expressions entirely.

   (a) $\left(\dfrac{x^2y^{-3}}{4y^{-1}z^3}\right)^{-2}$

   (b) $\dfrac{x^2-x-6}{6x-6} \div \dfrac{x^2+8x+12}{x^2-1}$

   (c) $\dfrac{x}{2x+4} - \dfrac{3}{x^2-4}$

2. [9] Determine whether the following are functions, and explain why or why not.

   (a) Temperature as a function of the time of day.

(b) Blood pressure as a function of number of final exams.

3. Solve the following inequalities for x. **Express your answer in interval notation, and draw a picture of the interval on the real line.**

   (a) [3] $-4 \leq 3x + 6 < 2$

   (b) [3] $-4x + 7 > 2x + 3$

   (c) [5] $x^2 + 7x < -12$

4. [25] Find equations for the functions fitting the following specifications.

   (a) The line going through the points $(2, 9)$ and $(2, 17)$.

   (b) The line going through the points $(-1, 3)$ and $(2, -1)$.

(c) The exponential function with initial value 20 that goes through the point $(1, 5)$.

(d) The exponential function going through $(0, 250)$ that has a growth rate of 60 %.

(e) The line perpendicular to $y = 3x + 2$ going through the point $(1, 2)$.

5. [6] Suppose a cellphone service charges \$40 per month for calls, texts, and up to 1GB of data, and an extra \$10 per GB of data used past the cap.

   (a) Write a piecewise function that gives $C(x)$, the cost of using x GB of data in a month.

   (b) Find and interpret $C(3.4)$.

6. [15] Suppose the German hamlet of Beispielburg has a population of 200 thousand people in the year 2000. In the year 2010, it has a population of 230 thousand people.

   (a) Find an exponential function that gives $P(t)$, the population of Beispielburg in thousands of people in terms of t, the number of years since 2000.

   (b) According to your model, what should the population of Beispielburg be in 2050?

   (c) When will the population reach 300 thousand people?

7. [6] Suppose the amount of studying done for an exam, in hours, is directly proportional to the grade received on the exam. Suppose also that one student studies for 3 hours and gets a 76 on their exam.

   (a) Find a function that gives the score on the exam as a function of the time spent studying.

   (b) Does your model make sense for all possible variables? Why or why not?

8. Solve the following equations for x.

(a) [3] $\frac{1}{2}(3x+4) = x - \frac{3}{2}$

(b) [3] $2x - 4 = 2$

(c) [5] $x^2 - 4x = 5$

(d) [5] $2x^2 = 5x - 3$

(e) [5] $2 \cdot 3^{x-4} = 8$

(f) [5] $4\log(2x-3) - 2 = 5$

9. [15] Let $f(x) = 3x + \frac{1}{x}$, $g(x) = e^{2x}$, and $h(x) = \ln(x) + x^2$. Find the following functions.

   (a) $(f \circ g)(x)$

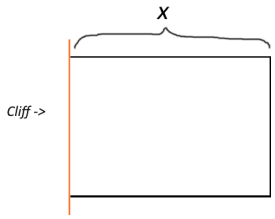
   (b) $(\frac{g}{h})(x)$

   (c) $(f - h)(x)$

   (d) The function obtained from shifting $f(x)$ to the right by 2 and up by 6.

   (e) The function obtained from reflecting $g(x)$ over the x-axis and stretching it by 3.

10. You knew it was coming. To complete her farm, Farmer O'Fencerty is building one last pen. The fence is being built as illustrated using a total of 500m of fencing.



(a) [5] Find a function for $A(x)$, the area of the pen in terms of x, the side adjacent to the cliff (as illustrated)

(b) [3] Find out what dimensions the area should have to maximise the area fenced in.

(c) [3] What is the maximum area?

11. [10] Suppose Yolanda has invested \$4000 in an account that gets 4% interest compounded monthly.

    (a) Find a function that gives $A(t)$, the amount of Yolanda's investment in terms of t, the number of years elapsed.

    (b) Find and interpret $A(30)$.

12. **Extra Credit:** Prove that the quadratic formula works. (Hint: Start with the form $ax^2 + bx + c$ and complete the square!).

13. **Extra Credit:** Prove that $\log(\frac{A}{B}) = \log(A) - \log(B)$ using other logarithm rules.

14. **Extra Credit:** What were your favourite and least favourite things about the class this semester?