Name:

Test 3: Ch 4 & 5

Complete the following problems to the best of your ability. **SHOW ALL OF YOUR WORK.** Unshown work will not be graded. You may use a calculator.

1. [12] Solve the following equations for x. You do not need to provide decimal approximations.

   (a) $-2 \cdot 3^{x-1} + 4 = 2$

   (b) $\log(4) + \log(x+1) = 2$

   (c) $\ln(2x+4) - 2 = 1$

2. [10] Simplify the following expressions.

   (a) $\log\left(\dfrac{10x^2}{\sqrt[3]{z-2}}\right)$

   (b) $\ln\left(\dfrac{x^3y^2}{3\sqrt{z}}\right)$

3. [15] Suppose there are 50 bacteria in a dish, and they double in population every hour.

   (a) Find a function $P(t)$ that models the population of bacteria at a time t hours after the beginning of the experiment.

   (b) Convert your function to an exponential function of base e. Find and interpret $P(5)$ using either function.

   (c) How long will it take for there to be 3000 bacteria in the dish?

4. Let $f(x) = -x + 1$, $g(x) = x^3$, $h(x) = \frac{1}{x}$, $j(x) = e^x$.

   (a) [4] Find $f \circ g(x)$.

   (b) [5] Find $h \circ f(x)$.

   (c) [5] Suppose $m(x) = j \circ f(x)$. Find $m^{-1}(x)$ (hint: write down what $m(x)$ is first, then find its inverse).

5. [10] Let $f(x) = x^3$.

   (a) Find the function that results from shifting $f(x)$ to the right by 2 and up by 3

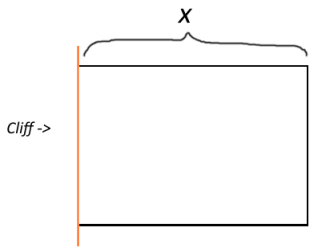
   (b) Find the function that results from scaling $f(x)$ by a factor of $1/2$ and then flipping it over the x-axis.

6. Let $g(x) = 2x^2 + 4x - 6$.

   (a) [5] Convert $g(x)$ into standard form.

   (b) [10] Sketch a graph of $g(x)$ below. Label the vertex and the y-intercept.

7. [15] Farmer O'Fencerty recently suffered an alien attack, so she has to build some of her fences over again. This time, she has 800 meters of fencing, and wants to build a pen around a nearby cliff.



(a) Find a function for $A(x)$, the area of the pen in terms of x, the side adjacent to the cliff (as illustrated)

(b) Find out what dimensions the area should have to maximise the area fenced in.

(c) What is the maximum area?

8. [15] Percival Fredrickstein Von Musel Klossowski de Rolo III invests 2000 gold pieces in a bank that promises 3.8% interest, compounded continuously.

(a) Find a function that gives A, the value of Percy's investment in gold pieces, as a function of t, the number of years elapsed.

(b) Find and interpret $A(10)$.

(c) How long will it take for Percy's investment to double in value (that is, to be worth 4000 gold pieces)?