Name:

HW 10: §4.5-5.4

Complete the following problems to the best of your ability. **SHOW ALL OF YOUR WORK.** Unshown work will not be graded. You may use a calculator.

1. Suppose a population of bacteria starts at 15 bacteria in a petri dish and that the population doubles every forty minutes.

   (a) Write a function that gives population in terms of time in hours since the start of the experiment.

   $A(t) = 15a^t$

   $a = 2^{\frac{3}{2}} = 2.83$

   $A(t) = 15(2.83)^t$

   (b) What is the population after 8 hours?

   $A(8) = 61713$ bacteria

   (c) How long does it take for the population to reach 1 million?

   $1000000 = 15(2.83)^t$

   $66666 = 2.83^t$

   $\log(66666) = \log(2.83^t) = t\log(2.83)$

   $t = \frac{\log(66666)}{\log(2.83)} \approx 10.68$

2. Solve for x.

   $\log(2x+1) + \log 2 = 2$

   $\log((2x+1)\cdot 2) = 2$

   $(2x+1)2 = 10^2$

   $4x + 2 = 100$

   $4x = 98$

   $x = 24.5$

3. Let $f(x) = 2x + 4$, $g(x) = e^x$, and $h(x) = -4x^3 + 3$

(a) Find $f \circ g(x)$

$= f(e^x) = 2e^x + 4$

(b) Find $g \circ h(x)$

$= g(-4x^3+3) = e^{-4x^3+3}$

(c) Find $f^{-1}(x)$

$y = 2x + 4$

$y - 4 = 2x \rightarrow \frac{y-4}{2} = x$

so $f^{-1}(x) = \frac{x-4}{2}$

4. Let $f(x) = x^2$. Write functions that represent the following geometric transformations.

(a) Shift up by 2.

$f(x) + 2 = x^2 + 2$

(b) Shift to the left by 6.

$f(x+6) = (x+6)^2$

(c) Stretch vertically by a factor of 2.

$2f(x) = 2x^2$

(d) Flip vertically and shrink by a factor of 2.

$-\frac{1}{2}f(x) = -\frac{1}{2}x^2$

5. Write the following quadratic functions in standard form.

(a) $x^2 + 4x - 3$

$x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - 3$

$= x^2 + 4x + 4 - 4 - 3$

$= (x+2)^2 - 7$

(b) $x^2 - 6x + 14$

$x^2 - 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 14$

$= x^2 - 6x + 9 - 9 + 14$

$= (x-3)^2 + 5$

6. Write the following quadratic functions in general form.

(a) $(x-4)^2 + 6$

$= x^2 - 8x + 16 + 6$

$= x^2 - 8x + 22$

(b) $(x+2)^2 - 3$

$= x^2 + 4x + 4 - 3$

$= x^2 + 4x + 1$

7. Let $f(x) = x^2 - 3x + 4$

(a) Express $f(x)$ in standard form.

$x^2 - 3x + 4$

$= x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 4$

$= x^2 - 3x + \frac{9}{4} - \frac{9}{4} + 4 = \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$

(b) Does f have a maximum or a minimum?

minimum, since it's a positive parabola

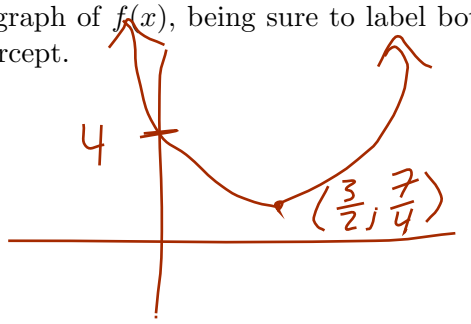
it's at $\left(\frac{3}{2}, \frac{7}{4}\right)$

(c) How many solutions does $f(x)$ have?

$b^2 - 4ac = 9 - 4(1)(4) = -7 < 0$

none.

(d) Sketch a graph of $f(x)$, being sure to label both the vertex and the y-intercept.



Optional Problems

4.5: All	4.6: 1-77	5.1: 1-18, 25-64
5.2: All	5.3: All	5.4: All