

MATH 172 - Mathematical Modelling for the Life Sciences - Spring 20

Problem Set 4

Here are some problems for you to try at home. Your quiz on February 21st will be based on the problems listed here, that is, if you can handle these problems, you can handle the quiz problem.

1. Giant pandas are a vulnerable species famous for their consumption of large amounts of bamboo. Pandas have three life stages: cubs (C), subadults (S) and reproductively mature adults (M). We have the following assumptions:
 - Cubs remain cubs for only one year, they have a mortality rate of 17%.
 - About 33% of subadult pandas mature each year. They have a mortality rate of 28%
 - A pair of mature pandas, on average, give birth to 2 pandas per year.
 - 97.7% of adults survive from one year to the next.
 - (a) Draw a transition diagram and the corresponding transition matrix for modelling this population of pandas.

- (b) If there are initially 10 cubs, 2 subadults and 4 adults, approximately how many pandas will there be in fifteen years?

2. Consider the matrix $A = \begin{bmatrix} 7 & 4 \\ -3 & -1 \end{bmatrix}$ and the vectors $\mathbf{v} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

- (a) Compute $A\mathbf{v}$ and $A\mathbf{w}$.

- (b) Compute $A^2\mathbf{v}$ and $A^2\mathbf{w}$.

- (c) What do you notice about your answers in (a) and (c)?

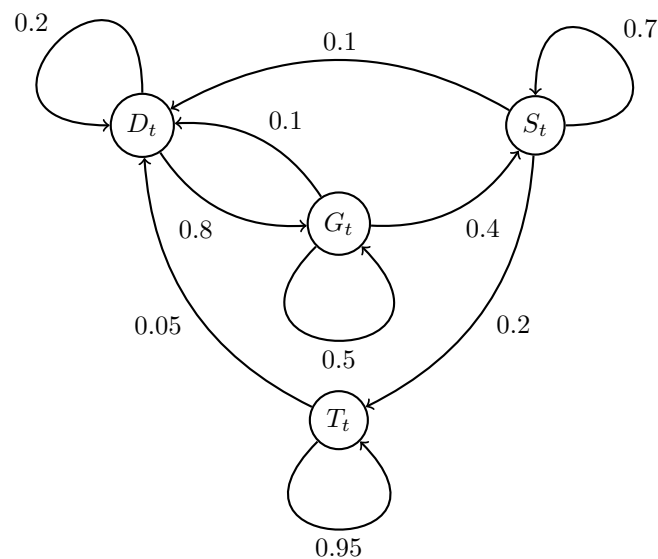
3. Consider the matrix $B = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$ and the vectors $\mathbf{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

(a) Compute $B\mathbf{v}$ and $B\mathbf{w}$.

(b) Compute $B^2\mathbf{v}$ and $B^2\mathbf{w}$.

(c) What do you notice about your answers in (a) and (c)?

4. An area of land is covered by dunes (D), grasses (G), shrubs (S) and trees (T). In this model, time is measured in decades. We have the following transition diagram:



We are looking at the model

$$\begin{bmatrix} D_t \\ G_t \\ S_t \\ T_t \end{bmatrix} = \mathbf{P}^t \begin{bmatrix} D_0 \\ G_0 \\ S_0 \\ T_0 \end{bmatrix}$$

For the questions that follow, round your answer to 3 decimal places.

(a) Write the matrix \mathbf{P} .

(b) If the current situation is 100% dunes, what will the distribution look like in two decades?

(c) If the current situation is 80% dunes and 20% shrubs, what will the distribution look like in two decades?

(d) If the current situation is

$$\begin{bmatrix} D_0 \\ G_0 \\ S_0 \\ T_0 \end{bmatrix} = \begin{bmatrix} 0.075 \\ 0.121 \\ 0.161 \\ 0.643 \end{bmatrix},$$

what is the distribution after one decade?

(e) Compare your answer in (d) to the initial conditions in (d).

(f) How might you describe (using language from the single population models of exam 1) the population totals given in the initial conditions of part (d)? What might you guess the habitat looks like in 1 million years?