Section 8.3: Trigonometric Integrals - Worksheet

Goal: By using trig identities combined with u-substitution, we'd like to find antiderivatives of the form

$$\int \sin^m(x)\cos^n(x)\,dx$$

(for integer values of m and n). The goal of this worksheet[1] is for you to work together in groups of 2-3 to discover the techniques that work for these anti-derivatives.

Example 1 - Warm-up: Find

$$\int \cos^4(x)\sin(x)\,dx.$$

Example 2: Find

$$\int \sin^3(x)\,dx.$$

(Hint: Use the identity $\sin^2(x)+\cos^2(x)=1$, then make a substitution.)

[1]Worksheet adapted from BOALA, `math.colorado.edu/activecalc`

Example 3: Find

$$\int \sin^5(x)\cos^2(x)\,dx.$$

(Hint: Write $\sin^5(x)$ as $\left(\sin^2(x)\right)^2 \sin(x)$.)

Example 4: Find

$$\int \sin^7(x)\cos^5(x)\,dx.$$

(The algebra here is long. Only set up the substitution - you do not need to fully evaluate.)

Example 5: In general, how would you go about trying to find

$$\int \sin^m(x)\cos^n(x)\,dx,$$

where m is odd? (Hint: consider the previous three problems.)

Example 6: Note that the same kind of trick works when the power on $\cos(x)$ is odd. To check that you understand, what trig identity and what u-substitution would you use to integrate

$$\int \cos^3(x)\sin^2(x)\,dx?$$

Example 7: Now what if the power on $\cos(x)$ and $\sin(x)$ are both even? Find

$$\int \sin^2(x)\,dx,$$

in each of the following two ways:

(a) Use the identity $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$.

(b) Integrate by parts, with $u = \sin(x)$ and $dv = \sin(x)\,dx$.

(c) Show that your answers to parts (a) and (b) above are the same by giving a suitable trig identity.

(d) How would you evaluate the integral

$$\int \sin^2(x)\cos^2(x)\,dx?$$

Example 8: Evaluate the integral in problem (2) above, again, but this time by parts using $u = \sin^2(x)$ and $dv - \sin(x)\,dx$. (After this, you'll probably need to do a substitution.)

Example 9 - For fun: Can you show your answers to problem (2) and (8) above are the same? It's another great trigonometric identity.

Example 10 - Further investigations: (especially for mathematics, physics and engineering majors) We also would like to be able to solve integrals of the form

$$\int \tan^m(x)\sec^n(x)\,dx.$$

These two functions play well with each other, since the derivative of $\tan(x)$ is $\sec^2(x)$, the derivative of $\sec(x)$ is $\sec(x)|tan(x)$ and since there is a Pythagorean identity relating them. It sometimes works to use $u = \tan(x)$ and it sometimes works to use $u = \sec(x)$. Based on the values of m and n, which substitution should you use? Are there cases for which neither substitution works? (See page 472 of the text.)