Section 8.2: Techniques of Integration

A New Technique: **Integration by parts** is a technique used to simplify integrals of the form

$$\int f(x)g(x)\,dx.$$

It is useful when one of the functions ($f(x)$ or $g(x)$) can be differentiated repeatedly and the other function can be integrated repeatedly without difficulty. The following are two such integrals:

$$\int x\cos(x)\,dx \text{ and } \int x^2e^x\,dx.$$

An Application of the Product Rule: If $f(x)$ and $g(x)$ are differentiable functions of x, the product rule says that

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x).$$

Integrating both sides and rearranging gives us the **Integration by Parts** formula!

In differential form, let $u = f(x)$ and $v = g(x)$. Then,

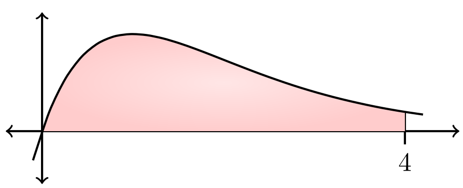
Integration by Parts Formula:

Remember, all of the techniques that we talk about are supposed to make integrating easier! Even though this formula expresses one integral in terms of a second integral, the idea is that the second integral, $\int v\,du$, is easier to evaluate. The key to integration by parts is making the right choice for u and v. Sometimes we may need to try multiple options before we can apply the formula.

Example 1: Find

$$\int x\cos(x)\,dx.$$

Example 3 - Integration by Parts for Definite Integrals: Find the area of the region bounded by the curve $y = xe^{-x}$ and the x-axis from $x = 0$ to $x = 4$.



Example 3: Evaluate

$$\int x^2 e^x \, dx.$$

Example 4 - Tabular Method: In Example 2 we have to apply the Integration by Parts Formula multiple times. There is a convenient way to "book-keep" our work. This is done by creating a table. Let's see how by examining Example 2 again.

Evaluate

$$\int x^2 e^x \, dx.$$

Example 5 - Recurring Integrals: Find the integral

$$\int e^x \sin(x)\, dx.$$

This "trick" comes up often when we are dealing with the product of two functions with "non-terminating" derivatives. By this we mean that you can keep differentiating functions like e^x and trig functions indefinitely and never reach 0. Polynomials on the other hand will eventually "terminate" and their n^{th} derivative (where n is the degree of the polynomial) is identically 0.

Example 6 - Challenge: Find the integral

$$\frac{1}{\pi}\int_0^{\pi} x^3 \cos(nx)\; dx,$$

where n is a positive integer.