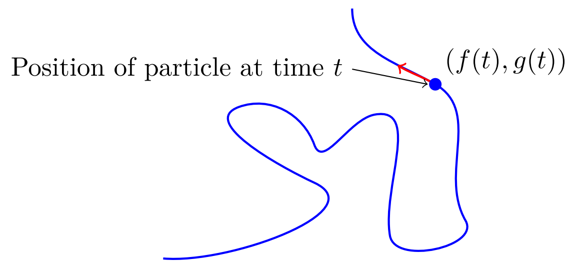
Section 11.1: Parametrisations of Plane Curves

Parametric Equations: Below we have the path of a moving particle on the xy-plane. We can sometimes describe such a path by a pair of equations, $x = f(t)$ and $y = g(t)$, where $f(t)$ and $g(t)$ are continuous functions. Equations like these describe more general curves than those described by a single function, and they provide not only the graph of the path traced out but also the location of the particle $(x, y) = (f(t), g(t))$ at any time t.



Definitions: If x and y are given as functions

$$x = f(t) \quad y = g(t),$$

over an interval I of t-values, then the set of points $(x, y) = (f(t), g(t))$ defined by these equations is a ____________________.

The equations are __________________________ for the curve.

The variable t is the ___________________ for the curve and its domain I is the _________________________.

If I is a closed interval, $a \leq t \leq b$, the ____________________ of the curve is the point $(f(a), g(a))$ and the

______________________ of the curve is $(f(b), g(b))$.

Example 1: Sketch the curve defined by the parametric equations

$$x = t^2, \quad y = t + 1, \quad -\infty < t < \infty.$$

Example 2: Identify geometrically the curve in Example 1 by eliminating the parameter t and obtaining an algebraic equation in x and y.

Example 3: Graph the parametric curves

(a) $x = \cos(t), \qquad y = \sin(t), \qquad 0 \leq t \leq 2\pi,$

(b) $x = a\cos(t), \quad y = a\sin(t), \quad 0 \leq t \leq 2\pi, \quad a \in \mathbb{R}.$

Example 4: The position $P(x, y)$ of a particle moving in the xy-plane is given by the equations and parameter interval

$$x = \sqrt{t}, \quad y = t, \quad t \geq 0.$$

Identify the path traced by the particle and describe the motion.

Example 5 - Natural Parametrisation: A parametrisation of the function $f(x) = x^2$ is given by

Example 6: Find a parametrisation for the line through the point (a, b) having slope m.

Example 7: Sketch and identify the path traced by the point $P(x, y)$ if

$$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}, \quad t > 0.$$