Math 142 **Name:** _____
Joseph C Foster **May 23rd, 2019**
Summer 2019 **Time Limit: 100 minutes**
Exam 1

This exam contains 6 pages (including this cover page) and 7 questions.
The total number of marks is 100. You have 100 minutes to complete the exam.
Read each question carefully. When specified, you must show **all** *necessary* work to receive full credit.
No calculators are allowed. Turn your phones off and place them on your desk face down. Remove any smartwatches and non-religious head-wear. Cheating of any kind will not be tolerated and will result in a grade of zero.

Question	Marks	Score	Question	Marks	Score
1	40		5	10	
2	10		6	10	
3	10		7	10	
4	10		Total	100	

Recitation									
Quizzes	1	2	3	4	5	6	7	8	9
	10	11	12	13	14	15	Average		
Midterms	Exam 1			Exam 2			Exam 3		
Final Exam									
Total									

1. Fill in the blanks to complete each of the following basic integration formulas.

(a) (2 marks) $\int \sec^2(x)\,dx =$ _____ $+C$

(b) (2 marks) $\int \csc^2(x)\,dx =$ _____ $+C$

(c) (2 marks) $\int \sec(x)\tan(x)\,dx =$ _____ $+C$

(d) (2 marks) $\int \csc(x)\cot(x)\,dx =$ _____ $+C$

(e) (2 marks) $\int \frac{1}{\sqrt{a^2-x^2}}\,dx \overset{(a>0)}{=}$ _____ $+C$

(f) (2 marks) $\int \frac{1}{a^2+x^2}\,dx \overset{(a>0)}{=}$ _____ $+C$

Fill in the blanks to complete the following angle-sum formulas. Your answer should involve trig functions with angles A and B.

(g) (2 marks) $\sin(A+B) =$ _____

(h) (2 marks) $\cos(A+B) =$ _____

Fill in the blanks to complete each statement/definition.

(i) (4 marks) **Trapezium Rule**: To approximate $\int_a^b f(x)\,dx$ using n trapeziums, use

$$T = \text{\_\_\_\_\_}.$$

where the y's are the values of f at the partition points

$$x_0 := a, \quad x_1 = a + \Delta x, \quad x_2 := a + 2\Delta x, \quad \ldots, \quad x_n := a + n\Delta x = b,$$

and $\Delta x =$ _____.

(j) (4 marks) **Error Estimates in the Trapezium Rule**: If $f''(x)$ is continuous and M is any upper bound for

the values of _____ on $[a,b]$, the the error E_T in the Trapezium Rule for approximating the

definite integral of $f(x)$ over the interval $[a,b]$ using n trapeziums satisfies the inequality

$$|E_T| \leq \text{\_\_\_\_\_}.$$

For parts (k)-(n), choose the best answer. Each part is worth 4 marks. There is only **one correct answer**, but you may choose up to **two answers**. If you choose two and one is the correct answer, you will receive 2 marks.

(k) (4 marks) When integrating $f(x) = x^3(x^4+1)^2$ with respect to x using u-substitution, we would let u equal:

☐ x^3

☐ $x(x^4+1)$

☐ x^4+1

☐ $(x^4+1)^2$

(l) (4 marks) If $x = 5\tan(\theta)$, then $\csc(\theta)$ is equal to:

☐ $\dfrac{5}{\sqrt{x^2+25}}$

☐ $\dfrac{x}{\sqrt{x^2+25}}$

☐ $\dfrac{\sqrt{x^2+25}}{5}$

☐ $\dfrac{\sqrt{x^2+25}}{x}$

(m) (4 marks) Which of the following is the correct partial fraction decomposition of $\dfrac{5x+20}{(x-3)(x+2)}$?

☐ $\dfrac{7}{x-3} - \dfrac{2}{x+2}$

☐ $\dfrac{2}{x-3} - \dfrac{7}{x+2}$

☐ $\dfrac{7}{x+2} - \dfrac{2}{x-3}$

☐ $\dfrac{2}{x+2} - \dfrac{7}{x-3}$

(n) (4 marks) Which of the following is an integral you cannot solve using trigonometric substitution?

☐ $\displaystyle\int \frac{1}{(1-x^2)^{3/2}}\,dx$

☐ $\displaystyle\int \frac{1}{\sqrt{x^4+1}}\,dx$

☐ $\displaystyle\int \frac{2x}{\sqrt{x^4+1}}\,dx$

☐ $\displaystyle\int \sqrt{x^2+1}\,dx$

For questions 2-7 show **all** *necessary* work to receive full credit. An answer with no work, even if correct, will not receive full marks. Please circle or box your final answer. If you cannot complete a problem but can write down what you want to do, and this is correct, you can still receive partial credit. Don't leave anything blank! The space provided is indicative of the amount of work required.

2. (10 marks) Compute

$$\int x^3 \sin(x)\, dx.$$

3. (10 marks) Evaluate

$$\int 35 \cos^4(x) \sin^3(x)\, dx.$$

4. (10 marks) Find

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx.$$

5. (10 marks) Compute

$$\int \frac{x^2+2x+6}{(x+1)(x^2+4)}\,dx.$$

6. (10 marks) Use the Trapezium Rule with $n = 4$ to approximate

$$\int_0^2 5x^2\,dx.$$

Your answer should be a single fraction with no sums. You do not have to put the fraction in lowest terms.

7. (10 marks) Decide whether

$$\int_0^3 \frac{1}{\sqrt{3-x}}\,dx$$

converges or diverges. If it converges, find the value of the integral.