Homework: Section 8.8

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1. $\int_0^{\infty} \frac{1}{x^2+1}\,dx = \lim_{t\to\infty} \int_0^t \frac{1}{x^2+1}\,dx = \lim_{t\to\infty} \tan^{-1}(x)\Big]_0^t$

$= \lim_{t\to\infty} \left(\tan^{-1}(t) - \tan^{-1}(0)\right) = \boxed{\pi/2}$ Converges.

3. $\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{t\to 0^+} \int_t^1 x^{-1/2}\,dx = \lim_{t\to 0^+} 2x^{1/2}\Big]_t^1$

$= \lim_{t\to 0^+} \left(2 - \underbrace{2t^{1/2}}_{\to 0}\right) = \boxed{2}$ Converges

5. $\int_{-1}^1 \frac{1}{x^{2/3}}\,dx = \int_{-1}^0 x^{-2/3}\,dx + \int_0^1 x^{-2/3}\,dx$

$= \lim_{a\to 0^-} \int_{-1}^a x^{-2/3}\,dx + \lim_{b\to 0^+} \int_b^1 x^{-2/3}\,dx$

$= \lim_{a\to 0^-} 3x^{1/3}\Big]_{-1}^a + \lim_{b\to 0^+} 3x^{1/3}\Big]_b^1$

$= \lim_{a\to 0^-} \left(3a^{1/3} + 3\right) + \lim_{b\to 0^+} \left(3 - 3b^{1/3}\right) = \boxed{6}$ Converges

7. $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx = \lim_{t\to 1^-} \int_0^t \frac{1}{\sqrt{1-x^2}}\,dx = \lim_{t\to 1^-} \sin^{-1}(x)\Big]_0^t$

$= \lim_{t\to 1^-} \left(\sin^{-1}(t) - \sin^{-1}(0)\right) = \boxed{\pi/2}$ Converges.

9. $\int_{-\infty}^{-2} \frac{2}{(x^2-1)}\,dx = \lim_{t\to -\infty} \int_t^{-2} \frac{2}{(x+1)(x-1)}\,dx = \lim_{t\to -\infty} \int_t^{-2} \left(\frac{-1}{x+1} + \frac{1}{x-1}\right)dx$

$\frac{2}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$

$2 = A(x-1) + B(x+1)$

$x=1 \quad 2 = B(2) \rightarrow B = 1$

$x=-1 \quad 2 = A(-2) \rightarrow A = -1$

$= \lim_{t\to -\infty} -\ln|x+1| + \ln|x-1|\Big]_t^{-2}$

$= \lim_{t\to -\infty} \ln\left|\frac{x-1}{x+1}\right|\Big]_t^{-2}$

$$= \lim_{t\to-\infty}\left(\ln 3 - \cancel{\ln\left|\frac{t-1}{t+1}\right|}^{\to 0}\right) = \boxed{\ln 3} \text{ Converges.}$$

11. $$\int_2^\infty \frac{2}{v^2-v}\,dv = \lim_{t\to\infty}\int_2^t \frac{2}{v(v-1)}\,dv$$

$$\frac{2}{v(v-1)} = \frac{A}{v} + \frac{B}{v-1}$$

$$2 = A(v-1) + B(v)$$

$v=1 \quad 2 = B$

$v=0 \quad 2 = A(-1) \to A=-2$

$$= \lim_{t\to\infty}\int_2^t \left(\frac{-2}{v} + \frac{2}{v-1}\right)dv$$

$$= \lim_{t\to\infty} -2\ln|v| + 2\ln|v-1|\Big]_2^t$$

$$= \lim_{t\to\infty} 2\ln\left|\frac{v-1}{v}\right|\Big]_2^t$$

$$= \lim_{t\to\infty} \cancel{2\ln\left|\frac{t-1}{t}\right|}^{\to 0} - 2\ln \tfrac{1}{2} = \boxed{2\ln 2} \text{ Converges.}$$

13. $$\int_{-\infty}^\infty \frac{2x}{(x^2+1)^2}\,dx = \int_{-\infty}^0 \frac{2x}{(x^2+1)^2}\,dx + \int_0^\infty \frac{2x}{(x^2+1)^2}\,dx$$

$$= \lim_{a\to-\infty}\int_a^0 \frac{2x}{(x^2+1)^2}\,dx + \lim_{b\to\infty}\int_0^b \frac{2x}{(x^2+1)^2}\,dx$$

$u = x^2+1$

$du = 2x\,dx$

$\int u^{-2}\,du = \frac{u^{-1}}{-1}$

$= \frac{-1}{x^2+1}$

$$= \lim_{a\to-\infty} \frac{-1}{x^2+1}\Big]_a^0 + \lim_{b\to\infty} \frac{-1}{x^2+1}\Big]_0^b$$

$$= \lim_{a\to-\infty}\left(-1 + \cancel{\frac{1}{a^2+1}}^{\to 0}\right) + \lim_{b\to\infty}\left(\cancel{\frac{-1}{b^2+1}}^{\to 0} + 1\right) = \boxed{0} \text{ Converges.}$$

15. $$\int_0^1 \frac{\theta+1}{\sqrt{\theta^2+2\theta}}\,d\theta = \lim_{t\to0^+}\int_t^1 \frac{\theta+1}{\sqrt{\theta^2+2\theta}}\,d\theta$$

$u = \theta^2+2\theta$

$du = (2\theta+2)\,d\theta$

$\frac{1}{2}du = (\theta+1)\,d\theta$

$\frac{1}{2}\int u^{-1/2}\,du = u^{1/2}$

$= (\theta^2+2\theta)^{1/2}$

$$= \lim_{t\to0^+}(\theta^2+2\theta)^{1/2}\Big]_t^1 = \lim_{t\to0^+}\left(3^{1/2} - \cancel{(t^2+2t)^{1/2}}^{\to 0}\right)$$

$$= \boxed{\sqrt{3}} \text{ Converges}$$

17. $\displaystyle\int_0^\infty \frac{1}{(1+x)\sqrt{x}}\,dx = \int_0^1 \frac{1}{(1+x)\sqrt{x}}\,dx + \int_1^\infty \frac{1}{(1+x)\sqrt{x}}\,dx$

$\displaystyle = \lim_{a\to 0^-} \int_a^1 \frac{1}{(1+x)\sqrt{x}}\,dx + \lim_{b\to\infty} \int_1^b \frac{1}{(1+x)\sqrt{x}}\,dx$

$\displaystyle = \lim_{a\to 0^-} 2\tan^{-1}(\sqrt{x})\Big]_a^1 + \lim_{b\to\infty} 2\tan^{-1}(\sqrt{x})\Big]_1^b$

$$\boxed{\begin{aligned} &U=\sqrt{x} \\ &dU = 1/2\,x^{-1/2}\,dx \\ &2dU = x^{-1/2}\,dx \\ &2\int \frac{1}{1+U^2}\,dU = 2\tan^{-1}(U) \\ &= 2\tan^{-1}(\sqrt{x}) \end{aligned}}$$

$\displaystyle = \lim_{a\to 0^-} \left(2\tan^{-1}(1) - 2\tan^{-1}(a)\right)$

$\displaystyle + \lim_{b\to\infty} \left(2\tan^{-1}(\sqrt{b}) - 2\tan^{-1}(1)\right)$

$= \cancel{2\pi/4} - 0 + 2(\pi/2) - \cancel{2\pi/4} = \boxed{\pi}$ Converges.

19. $\displaystyle\int_0^\infty \frac{1}{(1+v^2)(1+\tan^{-1}(v))}\,dv = \lim_{t\to\infty} \int_0^t \frac{1}{(1+v^2)(1+\tan^{-1}(v))}\,dv$

$\displaystyle = \lim_{t\to\infty} \ln|1+\tan^{-1}(v)|\Big]_0^t$

$$\boxed{\begin{aligned} &U = 1+\tan^{-1}(v) \\ &dU = \frac{1}{1+v^2}\,dv \\ &\int \frac{1}{U}\,dU = \ln|U| \\ &= \ln|1+\tan^{-1}(v)| \end{aligned}}$$

$\displaystyle = \lim_{t\to\infty} \ln|1+\tan^{-1}(t)| - \ln|1+\tan^{-1}(0)|$

$= \ln|1+\pi/2| - \cancel{\ln 1}^{\,0}$

$= \boxed{\ln|1+\pi/2|}$ Converges

21. $\displaystyle\int_{-\infty}^0 \theta e^\theta\,d\theta = \lim_{t\to-\infty} \int_t^0 \theta e^\theta\,d\theta = \lim_{t\to-\infty} \theta e^\theta - e^\theta \Big]_t^0$

$$\boxed{\begin{aligned} &U=\theta \quad dv = e^\theta d\theta \qquad \theta e^\theta - \int e^\theta d\theta \\ &dU = 1d\theta \quad v = e^\theta \qquad = \theta e^\theta - e^\theta \end{aligned}}$$

$\displaystyle = \lim_{t\to-\infty} -1 - (te^t - e^t)$

$\displaystyle = \lim_{t\to\infty} -1 - (t-1)e^t$

$$\lim_{t\to-\infty}(t-1)e^t = \lim_{t\to-\infty}\frac{t-1}{e^{-t}} \overset{L}{=} \lim_{t\to-\infty}\frac{1}{-e^{-t}} = 0$$

$$\Rightarrow \lim_{t\to-\infty} -1-(t-1)e^t = \boxed{-1} \text{ Converges.}$$

23. $$\int_{-\infty}^{0} e^{-|x|}dx = \lim_{t\to-\infty}\int_t^0 e^x dx = \lim_{t\to-\infty} e^x\Big]_t^0$$

$$= \lim_{t\to-\infty}(1-e^t) = \boxed{1}$$

$*$ on $(-\infty,0]$, $|x|=-x$

Converges.

25. $$\int_0^1 x\ln x\,dx = \lim_{t\to0^+}\int_t^1 x\ln x\,dx = \lim_{t\to0^+} \frac{1}{2}x^2\ln x - \frac{1}{4}x^2\Big]_t^1$$

$U=\ln x$ $\quad dV = x\,dx$

$dU = 1/x\,dx$ $\quad V=\frac{1}{2}x^2$

$$\frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx$$
$$= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2$$

$$= \lim_{t\to0^+}\left(0-\frac{1}{4}\right)-\left(\underset{*}{\frac{1}{2}t^2\ln t} - \frac{1}{4}t^2\right) = \boxed{-\frac{1}{4}} \text{ Converges}$$

$*$ $$\lim_{t\to0^+}\frac{1}{2}t^2\ln t = \lim_{t\to0^+}\frac{\frac{1}{2}\ln t}{t^{-2}} \overset{L}{=} \lim_{t\to0^+}\frac{\frac{1}{2t}}{-2t^{-3}} = \lim_{t\to0^+} -\frac{1}{4}\left(\frac{t^2}{1}\right) = 0$$

27. $$\int_0^2 \frac{ds}{\sqrt{4-s^2}} = \lim_{t\to2^-}\int_0^t\frac{ds}{\sqrt{4-s^2}} = \lim_{t\to2^-}\sin^{-1}(s/2)\Big]_0^t$$

$$\frac{1}{2}\int\frac{1}{\sqrt{1-s^2/4}}ds = \int\frac{1}{\sqrt{1-U^2}}dU$$

$U = s/2$ $\quad = \sin^{-1}(U)$

$dU = 1/2\,ds$

$2dU = ds$ $\quad = \sin^{-1}(s/2)$

$$= \lim_{t\to2^-}\left(\sin^{-1}(t/2) - \sin^{-1}(0)\right)$$

$$= \boxed{\pi/2} \text{ Converges.}$$