MATH 111*i* WORKSHEET 6

Name: Solutions

Work in groups to answer as many problems as you can. Ask questions if you get stuck.

1. Determine the average rate of change of the function between the given values of x.

(a) $f(x) = 3x + 2$, $a = x = 2$, $x = 5 = b$

$$\frac{3(5)+2-(3(2)+2)}{5-2} = \frac{17-8}{3} = \frac{9}{3}$$

Answer: 3

(b) $g(x) = 1 - x^2$, $a = x = 0$, $x = 1 = b$

$$\frac{1-(1)^2-(1-(0)^2)}{1-0} = \frac{0-1}{1} = \frac{-1}{1}$$

Answer: -1

(c) $h(x) = x^2 + 3x$, $a = x = -1$, $x = 1 = b$

$$\frac{(1)^2+3(1)-((-1)^2+3(-1))}{1-(-1)} = \frac{4-(-2)}{1+1} = \frac{4+2}{2} = \frac{6}{2}$$

Answer: 3

(d) $i(x) = 5 - 7x$, $a = x = -2$, $x = 3 = b$

$$\frac{5-7(3)-(5-7(-2))}{3-(-2)} = \frac{-16-19}{3+2} = \frac{-35}{5}$$

Answer: -7

(e) $j(x) = 0.5x^2 + 4$, $a = x = -2$, $x = 0 = b$

$$\frac{0.5(0)^2+4-(0.5(-2)^2+4)}{0-(-2)} = \frac{4-6}{0+2} = \frac{-2}{2} = -1$$

Answer: -1

(f) $k(x) = 2x + x^2$, $a = x = 2$, $x = 4 = b$

$$\frac{2(4)+(4)^2-(2(2)+(2)^2)}{4-2} = \frac{24-8}{2} = \frac{16}{2}$$

Answer: 8

2. In the latter part of the 20th century the united states experienced a large population shift from the cities to the suburbs. This is true of Atlanta, for example, whose population grew steadily for its first hundred years, then began to decline. Within the last two decades Atlanta's population has started to rise again, as seen in the table. Find the average rate of change of the population of Atlanta between the following years:

Year	Population	Year	Population
1850	2,572	1930	270,688
1860	9,554	1940	302,288
1870	21,789	1950	331,000
1880	37,409	1960	487,000
1890	65,533	1970	497,000
1900	89,872	1980	425,000
1910	154,839	1990	394,017
1920	200,616	2000	416,474

(a) 1850 and 1950

$$\frac{331{,}000-2572}{1950-1850} = \frac{328428}{100}$$

Answer: 100

(b) 1950 and 2000

$$\frac{416{,}474-331{,}000}{2000-1950} = \frac{383374}{50}$$

Answer: 50

(c) 1950 and 1970

$$\frac{497{,}000-331{,}000}{1970-1950} = \frac{166000}{20}$$

Answer: 20

3. Jason's height $H(x)$ (in inches) is a function of his age x (in years). At various stages in his life he grows at different rates, as shown in the table in the margin, which gives his height every year on his birthday.

Age	Height	Age	Height	Age	Height
0	19.25	6	44.50	15	66.50
1	28.00	8	49.35	16	69.13
2	32.50	10	54.38	17	69.50
3	36.25	12	58.75	18	69.75
4	39.63	14	64.00	19	69.88

(a) Find the average rate of change in Jason's height from birth to 4 years.

$\frac{39.63 - 19.25}{4 - 0} = \frac{20.38}{4} =$

Answer: 5.095

(b) Find the average rate of change in Jason's height from ages 12 to 17.

$\frac{69.50 - 58.75}{17 - 12} = \frac{10.75}{5} =$

Answer: 2.15

Challenge: Between what years did Jason grow the fastest?

Answer: 0 and 1

4. Weather ballons are filled with hydrogen and released at various sites to measure and transmit data such as air pressure and temperature. A weather balloon is filled with hydrogen at a rate of $0.5\text{ft}^3/\text{s}$. Initially, the balloon has 2ft^3 of hydrogen. Find a linear function that models the volume of hydrogen in the balloon after t seconds.

m b

Answer: $y = 0.5t + 2$

5. A large koi pond is filled from a garden hose at the rate of 10 gal/min. Initially, the pond contains 300 gallons of water. Find a linear function that models the volume of water in the pond after t minutes.

m b

Answer: $y = 10t + 300$

6. A local diner must build a wheelchair ramp to provide handicap access to their restaurant. Federal building codes require that a wheelchair ramp must have a maximum rise of 1 inch for every horizontal distance of 12 inches. What is the maximum allowable *slope* for a wheelchair ramp.

$m = \frac{\text{rise}}{\text{run}}$

Answer: $m \leq \frac{1}{12}$.

Cont.

7. Find an equation of the line with the given slope and y-intercept.

(a) Slope 5, y-intercept 2

Answer: $y = 5x + 2$

(b) Slope -1, y-intercept -3

Answer: $y = -x - 3$

(c) Slope $\frac{1}{2}$, y-intercept 5

Answer: $y = \frac{1}{2}x + 5$

(d) Slope 2, y-intercept 7

Answer: $y = 2x + 7$

(e) Slope -5, y-intercept -1

Answer: $y = -5x - 1$

(f) Slope $-\frac{1}{2}$, y-intercept $-\frac{1}{4}$

Answer: $y = -\frac{1}{2}x - \frac{1}{4}$

8. Find an equation of the line with the given slope that passes through the given point. Give your answer in both point-slope and slope-intercept form.

(a) Slope 2, through $(0, 4)$

Slope Point

Answer: $y - 4 = 2(x - 0)$

Answer: $y = 2x + 4$

(b) Slope $\frac{2}{3}$, through $(1, 7)$

Answer: $y - 7 = \frac{2}{3}(x - 1)$

Answer: $y = \frac{2}{3}x + \frac{19}{3}$

(c) Slope $-\frac{1}{3}$, through $(-6, 4)$

Answer: $y - 4 = -\frac{1}{3}(x - (-6))$

Answer: $y = -\frac{1}{3}x + 2$

(d) Slope 0, through $(4, -5)$

Answer: $y - (-5) = 0(x - 4)$

Answer: $y = -5$

(e) Slope -3, through $(0, -2)$

Answer: $y - (-2) = -3(x - 0)$

Answer: $y = -3x - 2$

(f) Slope $\frac{3}{2}$, through $(0, -2)$

Answer: $y - (-2) = \frac{3}{2}(x - 0)$

Answer: $y = \frac{3}{2}x - 2$

(g) Slope $-\frac{3}{4}$, through $(-4, -3)$

Answer: $y - (-3) = -\frac{3}{4}(x - (-4))$

Answer: $y = -\frac{3}{4}x - 6$

(h) Slope 0, through $(-1, 1)$

Answer: $y - 1 = 0(x - (-1))$

Answer: $y = 1$.

Cont.

9. Find an equation of the line that passes through the two given points. Give your answer in both point-slope and slope-intercept form.

(a) $(-2,1)$ and $(4,7)$

$m = \frac{7-1}{4-(-2)} = \frac{6}{4+2} = \frac{6}{6} = 1$

Answer: $y-7=1(x-4)$

Answer: $y=x+3$

(b) $(-1,7)$ and $(2,-2)$

$m = \frac{-2-7}{2-(-1)} = \frac{-9}{2+1} = \frac{-9}{3} = -3$

Answer: $y-(-2)=-3(x-2)$

Answer: $y=-3x+4$

(c) $(2,3)$ and $(5,7)$

$m = \frac{7-3}{5-2} = \frac{4}{3}$

Answer: $y-7=\frac{4}{3}(x-5)$

Answer: $y=\frac{4}{3}x+\frac{1}{3}$

(d) $(-1,6)$ and $(1,2)$

$m = \frac{2-6}{1-(-1)} = \frac{-4}{1+1} = \frac{-4}{2} = -2$

Answer: $y-2=-2(x-1)$

Answer: $y=-2x+4$

(e) $(-1,-1)$ and $(3,7)$

$m = \frac{7-(-1)}{3-(-1)} = \frac{7+1}{3+1} = \frac{8}{4} = 2$

Answer: $y-7=2(x-3)$

Answer: $y=2x+1$

(f) $(2,-1)$ and $(5,3)$

$m = \frac{3-(-1)}{5-2} = \frac{3+1}{3} = \frac{4}{3}$

Answer: $y-3=\frac{4}{3}(x-5)$

Answer: $y=\frac{4}{3}x-\frac{11}{3}$

(g) $(5,2)$ and $(-2,3)$

$m = \frac{3-2}{-2-5} = \frac{1}{-7} = -\frac{1}{7}$

Answer: $y-3=-\frac{1}{7}(x-(-2))$

Answer: $y=-\frac{1}{7}x+\frac{19}{7}$

(h) $(7,6)$ and $(4,-2)$

$m = \frac{-2-6}{4-7} = \frac{-8}{-3} = \frac{8}{3}$

Answer: $y-6=\frac{8}{3}(x-7)$

Answer: $y=\frac{8}{3}x-\frac{38}{3}$

Cont.

10. Air traffic controllers at most airports use a radar system to identify the speed, position, and other information about approaching aircraft. Using radar, an air traffic controller identifies an approaching aircraft and determines that it is 45 miles away from the radar tower. five minutes later, she determines that the aircraft is 25 miles from the radar tower. Assume that the aircraft is approaching the radar tower directly at a constant speed.

(a) What is the speed of the approaching aircraft?

distance = 45 - 25 = 20
time = 5

Answer: $\frac{20}{5} = 4$ miles/minute

(b) Find a linear equation that models the distance y of the aircraft from the radar tower x minutes after it was first observed.

Speed = 4
distance = decreasing
initial = 45

Answer: $D = -4t + 45$

11. A small business owner buys a truck for \$25,000 to transport supplies for her business. She anticipates that she will use the truck for 5 years and that the truck will be worth \$10,000 in 5 years.

(a) At what rate is the truck depreciating at?

Drop in value = 25000 - 10000 = 15000
time = 5

Answer: $\frac{15000}{5} = 3000$ \$/year

(b) Find a linear model that represents the depreciated value of the truck t years after it was purchased.

rate = 3000
value = decreasing
initial = 25000

Answer: $V = -3000t + 25000$

12. The manager of a furniture factory finds that the cost of manufacturing chairs depends linearly on the number of chairs produced. It costs \$2200 to make 100 chairs and \$4800 to make 300 chairs.

(a) Find a linear equation that models the cost y of making x chairs.

$m = \frac{4800 - 2200}{300 - 100} = \frac{2600}{200} = 13$

$y - 2200 = 13(x - 100)$

$y = 13x - 1300 + 2200$

Answer: $y = 13x + 900$

(b) What does the y-intercept of the line represent?

Initial Cost

(c) What does the slope of the line represent?

Cost per chair.

Cont.

13. Express the given equation in slope-intercept form.

(a) $3x + y = 6$

Answer: $y = 6 - 3x$

(b) $9x - 3y - 4 = 0$

$9x - 4 = 3y$

Answer: $y = 3x - 4/3$

(c) $4y + 5x = 10$

$4y = -5x + 10$

Answer: $y = -\frac{5}{4}x + \frac{5}{2}$

(d) $x + 4y = 10$

$4y = -x + 10$

Answer: $y = -\frac{1}{4}x + \frac{5}{2}$

(e) $2x - 8y + 5 = 0$

$2x + 5 = 8y$

Answer: $y = \frac{1}{4}x + \frac{5}{8}$

(f) $4y + 8 = 0$

$4y = -8$

Answer: $y = -2$

14. Find an equation of the line that satisfies the given conditions.

(a) Through $(-1, 2)$, parallel to the line $y = 4x + 7$

$m = 4$

Answer: $y - 2 = 4(x + 1)$ or $y = 4x + 6$

(b) Through $(2, 3)$, parallel to the line $y = -2x - 5$

$m = -2$

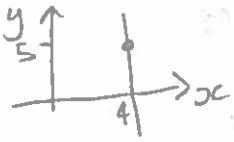
Answer: $y - 3 = -2(x - 2)$ or $y = -2x + 7$

(c) Through $(2, 2)$, parallel to the line $y = 5$

$m = 0$

Answer: $y - 2 = 0$ or $y = 2$

(d) Through $(4, 5)$, parallel to the y-axis



Answer: $x = 4$

(e) Through $(3, -3)$, parallel to the line $x = -1$

as above

Answer: $y = -3$

(f) Through $(4, 5)$, parallel to the x-axis

$m = 0$

Answer: $y - 5 = 0$ or $y = 5$

Cont.

(g) Through $(2, 6)$, perpendicular to the line $y = \frac{2}{3}x + 5$

$m = -3/2$

Answer: $y - 6 = -3/2(x-2)$ or $y = -3/2x + 9$

(h) Through $(-6, 6)$, perpendicular to the line $y = \frac{3}{4}x - 2$

$m = -4/3$

Answer: $y - 6 = -4/3(x+6)$ or $y = -4/3x - 2$

(i) Through $(5, -\frac{1}{5})$, perpendicular to the line $4y = 5x - 3$

$y = 5/4x - 3/4$ $\quad m = -4/5$

Answer: $y + 1/5 = -4/5(x-5)$ or $y = -4/5x + 19/5$

(j) Through $(\frac{1}{2}, -\frac{1}{4})$, perpendicular to the line $4x - 8y = 1$

$y = 1/2x - 1/8$, $\quad m = -2$

Answer: $y + 1/4 = -2(x - 1/2)$ or $y = -2x + 3/4$.

15. Mauricio and Thanh are kayaking south down a river heading toward some rapids. Mauricio leaves 45 miles north of the rapids at 6 : 00 a.m., and Thanh leaves 24 miles north of the rapids at 8 a.m.. Both boys maintain a constant speed of 5mi/h.

(a) For each boy, find a linear equation that relates their distance y from the rapids at time x (Take $x = 0$ to be 6 : 00 a.m., so 8 : 00 a.m. would be $x = 2$).

Mauricio
speed = 5
distance = decreasing
$x = 0 \Rightarrow M = 45$

Tanh:
speed = 5
distance = decreasing
$x = 2 \Rightarrow T = 24$
} $T - 24 = -5(x-2)$

Answer: $M = -5x + 45$

Answer: $T = -5x + 34$

(b) Will Mauricio ever pass Thanh?

Answer: No.

To see why:
Suppose he does. Then there exists an x when M reaches T. So,

$M = T$
$\Rightarrow -5x + 45 = -5x + 34$
$\Rightarrow 45 = 34$ which is clearly nonsense.

Cont.

16. Kathie and her friend Tia are on their motorcycles heading north to Springfield on the same straight highway. Kathie leaves from a point 120 miles south of Springfields at 10 : 00 a.m., and Tia leaves from a point 35 miles south of Springfield at 11 : 00 a.m.. Both girls maintain a constant speed of 75mi/h.

(a) For each girl, find a linear equation that relates her distance y from Springfield at time x. (Take $x = 0$ to be 10 : 00 a.m., so 11 : 00a.m. would be $x = 1$).

Kathie
Speed = 75
distance = decreasing
$x=0 \Rightarrow K=120$

Tia
Speed = 75
distance = decreasing
$x=1 \Rightarrow T=35$
$T-35=-75(x-1)$

Answer: $K=-75x+120$

Answer: $T=-75x+110$

(b) Will Kathie ever pass Tia?

Same reasoning as in 15

Answer: No

17. The amount of interest i earned from a CD is directly proportional to the amount of money P invested in the CD. Hiam invests \$1500 in a 12−month CD and earns \$90.00 in interest at maturity. Find the equation of proportionality that relates i to P.

$i = kP$

$90 = k \times 1500$

$k = \frac{90}{1500} = \frac{3}{50}$

Answer: $i = \frac{3}{50}P$

18. Perry invests in a high-yield money market account that has an APY of 6.17%. This means that when the effects of compounding are included, Perry's investment yields 6.17% each year.

(a) Find the equation of proportionality that relates the amount of interest i earned in one year to the amount of the investment P.

$i = kP$

$6.17 = k \times 100$

$k = \frac{6.17}{100} = \frac{617}{10{,}000}$

Answer: $i = \frac{617}{10{,}000}P$

(b) If Perry invests \$2500, what is the amount of interest that the investment earns in one year?

$i = \frac{617}{10{,}000} \cdot 2500$

Answer: $\$\frac{617}{4}$

Cont.

19. A balloon is being filled with air. The linear equation

$$V = 2 + 0.05t$$

models the volume V (in cubic feet) of air in the balloon at any time t (in seconds). How many minutes will it take until the balloon contains $55ft^3$ of hydrogen?

$55 = 2 + 0.05t$
$53 = 0.05t$
$53 = \frac{5}{100}t$

$t = \frac{53 \cdot 100}{5} = 53 \cdot 20$ seconds

$\frac{t}{60} = \frac{53 \cdot 20}{60} = \frac{53}{3}$ minutes

Answer: _____

20. An aircraft is approaching an international airport. Using radar, an air traffic controller determines that the linear equation

$$y = -4x + 45$$

models the distance (measured in miles) of the approaching aircraft from the radar tower x minutes since the radar identified the aircraft. How many minutes will it take for the aircraft to reach the radar tower?

$0 = -4x + 45$
$4x = 45$

Answer: 45/4 minutes.

21. Biologists have observed that the chirping rate of a certain species of cricket is modeled by the linear equation

$$t = \tfrac{5}{24}n + 45$$

where t is the temperature (in degrees Fahrenheit) and n is the number of chirps per minute. If the temperature is $80°F$, estimate the cricket's chirping rate.

$80 = \frac{5}{24}n + 45$
$35 = \frac{5}{24}n$

$n = \frac{35 \cdot 24}{5} = 7 \cdot 24$

Answer: 168

22. A graphic artist needs to construct a design that uses a rectangle whose length is 5cm longer that its width x.

(a) Construct a model that gives the perimeter of the rectangle

$P = x + x + (x+5) + (x+5)$

Answer: $P = 4x + 10$

(b) If the perimeter of the rectangle is 26cm, what are the dimensions of the rectangle

$26 = 4x + 10$
$16 = 4x$
$x = 4$
$x + 4 = 8$

Answer: 4 × 8.

The End.