Solutions

Name:_____

> Work in groups to answer as many problems as you can. Ask questions if you get stuck.

1. Find a linear model to represent the data for each of the following tables.

(a)

x	y
0	-10
1	-4
2	2
3	8

Answer: $y = 6x - 10$

(d)

x	y
0	110
2	98
4	86
6	74

Answer: $y = 110 - 6x$

(b)

u	v
0	205
1	218
2	231
3	244

Answer: $v = 205 + 13u$

(e)

w	z
0	55
3	52
6	49
9	46

Answer: $z = 55 - w$

(c)

a	b
0	17
3	38
6	59
9	80

Answer: $b = 17 + 7x$

(f)

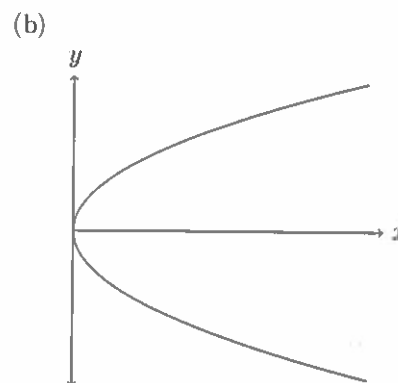| x	y
0	19
1.5	34
3	49
4.5	64

Answer: $y = 19 + 10x$

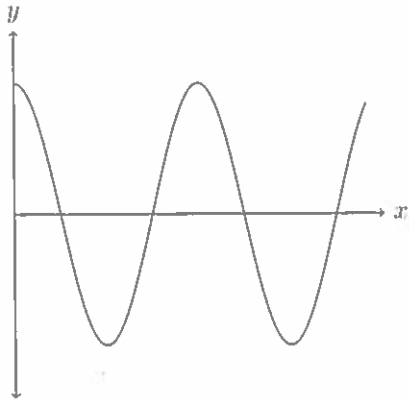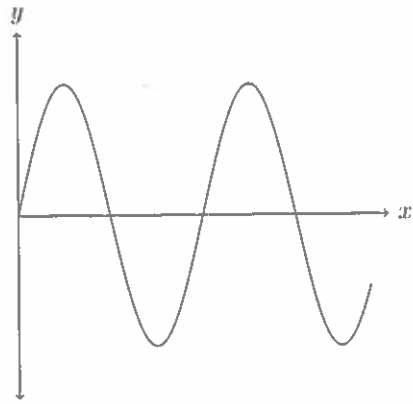2. Determine if the graphs below represent functions.
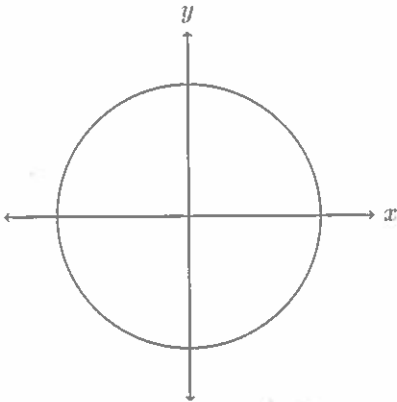
(a)

Answer: Yes

(b)
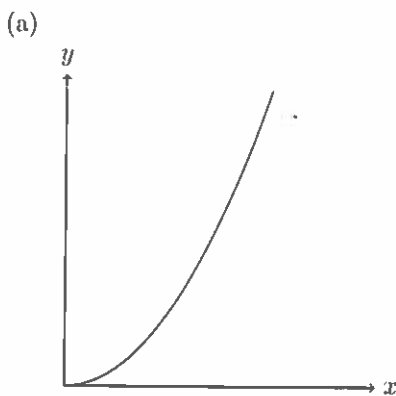
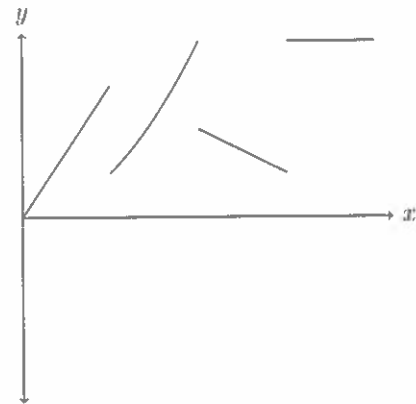Answer: No

(c)
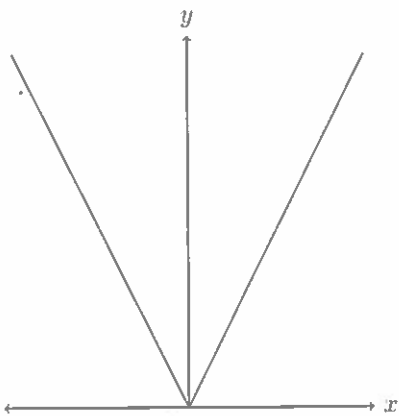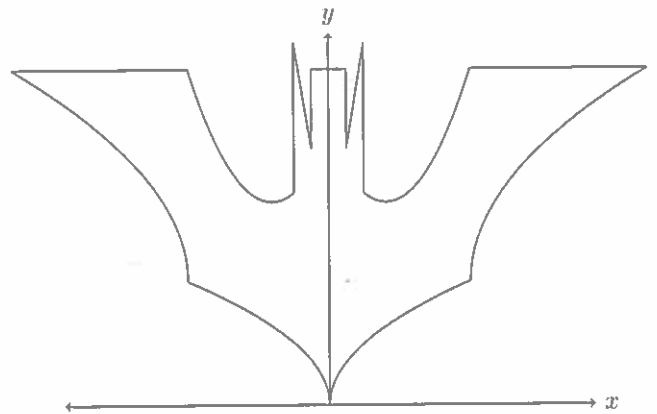

Answer: Yes

(f)

Answer: Yes

(d)

Answer: No

(g)

Answer: Yes

(e)

Answer: Yes

(h)

Answer: No

3. Find the domain of the given function.

(a) $f(x) = 9 - x$

Answer: all real numbers

(b) $P(z) = z^2 - 4$

Answer: all real numbers

(c) $h(x) = \dfrac{2 + x}{8x - 1}$

Answer: $x \neq \frac{1}{8}$

(d) $A(t) = \dfrac{t^2 - 4}{t^2 + 6t - 7}$

$= \dfrac{(t+2)(t-2)}{(t+7)(t-1)}$

Answer: $t \neq -7, 1$

(e) $h(w) = \dfrac{w^2 + 3w + 2}{w^2 + 12w + 36}$

$= \dfrac{(w+1)(w+2)}{(w+6)^2}$

Answer: $w \neq -6$

(f) $g(x) = \sqrt{10x - 15}$

$10x - 15 \geqslant 0$

Answer: $x \geqslant \frac{3}{2}$

(g) $f(t) = \dfrac{10t}{\sqrt{6 - 4t}}$

$6 - 4t > 0$

Answer: $t < \frac{3}{2}$

(h) $f(w) = \dfrac{\sqrt{w + 7}}{\sqrt{2 - w}}$ $\begin{cases} 2 - w > 0 \\ w + 7 \geqslant 0 \end{cases}$

Answer: $-7 \leqslant w < 2$

(i) $A(z) = \sqrt{z^2 - 9z}$

$z^2 - 9z \geqslant 0$

$z(z - 9) \geqslant 0$

Answer: $0 \leqslant z \leqslant 9$

(j) $h(z) = \sqrt{z^2 - z - 20}$

$z^2 - z - 20 \geqslant 0$

$(z - 5)(z + 4) \geqslant 0$

Answer: $z \leqslant -4$
         $z \geqslant 5$

(k) $g(t) = \sqrt{\dfrac{6 + t}{5t - 10}}$

$\begin{cases} 6 + t \geqslant 0 \Rightarrow t \geqslant -6 \\ 5t - 10 > 0 \Rightarrow t > 2 \end{cases}$

Answer: $t > 2$

(l) $g(t) = \sqrt{\dfrac{6 + t}{10 - 5t}}$

$\begin{cases} 6 + t \geqslant 0 \Rightarrow t \geqslant -6 \\ 10 - 5t > 0 \Rightarrow t < 2 \end{cases}$

Answer: $-6 \leqslant t < 2$

Cont.

4. (a) An internet bookstore charges \$15 shipping for orders under \$100 but provides free shipping for orders of \$100 or more.

     i. Find a piecewise function C that represents the cost of an order based on the total price x of books purchased.

$$C(x) = \begin{cases} x+15, & x < 100 \\ x, & x \geq 100 \end{cases}$$

     ii. Find $C(75)$, $C(100)$ and $C(105)$.

Answer: $90

Answer: $100

Answer: $105

(b) A hotel chain charges \$74 dollars per night for the first two nights and \$50 for each additional night.

     i. Express the total cost T as function of the number of nights x that a guest stays.

$$T(x) = \begin{cases} 74x, & x \leq 2 \\ 148 + 50(x-2), & x > 2 \end{cases}$$

     ii. Find $T(2)$, $T(3)$ and $T(5)$.

Answer: $148

Answer: $198

Answer: $298

(c) In a certain state, the maximum speed permitted on freeways is 65mph, and the minimum is 40mph. The fine for violating these limits is $15 for every mile above the speed limit or $10 for every mile below the speed limit.

   i. Express the total cost of the fine F in terms of the driving speed x on the freeway.

$$F(x) = \begin{cases} (40-x)10, & x < 40 \\ 0 & 40 \le x \le 65 \\ (x-65)15 & 65 < x \end{cases}$$

   ii. Find $F(30)$, $F(50)$ and $F(75)$.

Answer:_____$100_____

Answer:_____$0_____

Answer:_____$150_____

(d) A utility company chatdes a base rate of 10 cents per kilowatt hour (kWh) for the first $350kWh$ and 15 cents per kilowatt hour for all additional electricity usage.

   i. Express the amount E that the utility company charges when x kilowatt hours of electricity is used.

$$E(x) = \begin{cases} 0.1x, & x \le 350 \\ 35 + 0.15(x-350), & x > 350 \end{cases}$$

   ii. Find $E(300)$, $E(350)$ and $E(600)$.

Answer:_____$30_____

Answer:_____$35_____

Answer:_____$72.5_____

(e) To discourage excessive water use, a city charges its residents $0.008 per gallon for households that use less than 4,000 gallons a month and $0.012 for households that use 4,000 gallons or more.

　　i. Find a piecewise function C that gives the water bill for a household using x gallons of water.

$$C(x) = \begin{cases} 0.008x & x < 4{,}000 \\ 0.012x & x \geq 4{,}000 \end{cases}$$

　　ii. Find $C(3,900)$ and $C(4,200)$.

Answer:　$31.20

Answer:　$50.40

(f) In a certain country, income tax T is assessed based on a persons total income (in dollars). A person is charged as follows;

- If the persons income is $10,000 or less, no tax is charged.
- If the person earns more than $10,000 but no more than $20,000, they are charged 8% of their total income.
- If the person earns more than $20,000, they are charged the same %8 on anything up to $20,000 and a further 15% on anything over 20,000.

　　i. Find a piecewise function T that represents the tax paid by a person earning x dollars.

$$T(x) = \begin{cases} 0 & x \leq 10{,}000 \\ 0.08x, & 10{,}000 < x \leq 20{,}000 \\ 1600 + 0.15(x - 20{,}000), & 20{,}000 < x. \end{cases}$$

　　ii. Find $T(5,000)$, $T(12,000)$ and $T(25,000)$.

Answer:　$0

Answer:　$960

Answer:　$2350

The End.