MATH 111i HOMEWORK 8 DUE THURSDAY, NOVEMBER 16TH

Name: Solutions

This assignment is worth 100 points. You will be awarded 40 points for attempting the entire assignment (that is answer all problems). A random selection of problems will be graded for the remaining 60 points. The space left between each question is indicative of how much work you should show. If there are any problems you find particularly difficult, circle them in red. If there are any particular questions you would like feedback on, circle them in green. These are examples of questions that might appear on an exam/quiz. If you use a calculator to help, make sure you can also do them without it.

1. Find the inverses of the following functions.

(a) $f(x) = 12x - 7$

$y = 12x - 7$

$y + 7 = 12x$

$x = \frac{y+7}{12}$

Answer: $f^{-1}(x) = \frac{x+7}{12}$

(b) $f(x) = 7x$

$y = 7x$

$y/7 = x$

Answer: $f^{-1}(x) = x/7$

(c) $f(x) = \frac{3}{4} - \frac{9}{7}x$

$y = \frac{3}{4} - \frac{9}{7}x$

$\frac{9}{7}x = \frac{3}{4} - y$

$\frac{9}{7}x = \frac{3-4y}{4}$

$x = \frac{7}{9} \cdot \frac{3-4y}{4}$

Answer: $f^{-1}(x) = \frac{7(3-4x)}{36}$

(d) $f(x) = (1-4x)^3$

$y = (1-4x)^3$

$y^{1/3} = 1 - 4x$

$1 - y^{1/3} = 4x$

Answer: $f^{-1}(x) = \frac{1-x^{1/3}}{4}$

(e) $f(x) = 4 - (x+3)^5$

$y = 4 - (x+3)^5$

$(x+3)^5 = 4 - y$

$x + 3 = \sqrt[5]{4-y}$

$x = \sqrt[5]{4-y} - 3$

Answer: $f^{-1}(x) = (4-x)^{1/5} - 3$

(f) $f(x) = \sqrt[7]{5-8x}$

$y = \sqrt[7]{5-8x}$

$y^7 = 5 - 8x$

$8x = 5 - y^7$

Answer: $f^{-1}(x) = \frac{5-x^7}{8}$

(g) $f(x) = \frac{10-3x}{8x}$

$y = \frac{10-3x}{8x}$

$8xy = 10 - 3x$

$8xy + 3x = 10$

$x(8y+3) = 10$

$x = \frac{10}{8y+3}$

Answer: $f^{-1}(x) = \frac{10}{8x+3}$

(h) $f(x) = \frac{6x-7}{4+x}$

$y = \frac{6x-7}{4+x}$

$y(4+x) = 6x - 7$

$4y + xy = 6x - 7$

$4y + 7 = 6x - xy$

$4y + 7 = x(6-y)$

$\frac{4y+7}{6-y} = x$

Answer: $f^{-1}(x) = \frac{4x+7}{6-x}$

2. Verify that $f(x)$ and $g(x)$ are both inverses of each other by, either, computing $f(g(x))$ and $g(f(x))$ or finding the inverse of one of them and showing it is equivalent to the other.

(a) $f(x) = 27x^3 + 1$, $g(x) = \frac{1}{3}\sqrt[3]{x-1}$

$y = 27x^3 + 1$
$y - 1 = 27x^3$
$(y-1)^{1/3} = 3x$
$\frac{1}{3}(y-1)^{1/3} = x$
$f^{-1}(x) = \frac{1}{3}\sqrt[3]{x-1}$

or

$f(g(x)) = 27g(x)^3 + 1$
$= 27\left(\frac{1}{3}\sqrt[3]{x-1}\right)^3 + 1$
$= 27 \cdot \frac{1}{27}(x-1) + 1$
$= x - 1 + 1$
$= x$

$g(f(x)) = \frac{1}{3}\sqrt[3]{f(x) - 1}$
$= \frac{1}{3}\sqrt[3]{27x^3 + 1 - 1}$
$= \frac{1}{3}\sqrt[3]{27x^3}$
$= \frac{1}{3}3x$
$= x$

(b) $f(x) = \ln\left(\frac{1}{2}x - 7\right)$, $g(x) = 2e^x + 14$

$y = 2e^x + 14$
$y - 14 = 2e^x$
$\frac{1}{2}y - 7 = e^x$
$\ln(\frac{1}{2}y - 7) = x$
$g^{-1}(x) = \ln(\frac{1}{2}x - 7)$

or

$f(g(x)) = \ln(\frac{1}{2}g(x) - 7)$
$= \ln(\frac{1}{2}(2e^x + 14) - 7)$
$= \ln(e^x + 7 - 7)$
$= \ln(e^x)$
$= x$

$g(f(x)) = 2e^{f(x)} + 14$
$= 2e^{\ln(\frac{1}{2}x - 7)} + 14$
$= 2(\frac{1}{2}x - 7) + 14$
$= x - 14 + 14$
$= x$

(c) $f(x) = 3^{5x-1}$, $g(x) = \frac{1}{5}\log_3(3x)$

$y = \frac{1}{5}\log_3(3x)$
$5y = \log_3(3x)$
$5y = \log_3(x) + 1$
$5y - 1 = \log_3(x)$
$g^{-1}(x) = 3^{5x-1}$

or

$f(g(x)) = 3^{5g(x) - 1}$
$= 3^{\log_3(3x) - 1}$
$= \frac{1}{3}3^{\log_3(3x)}$
$= \frac{1}{3}3x$
$= x$

$g(f(x)) = \frac{1}{5}\log_3(3f(x))$
$= \frac{1}{5}\log_3(3 \cdot 3^{5x-1})$
$= \frac{1}{5}\log_3(3^{5x})$
$= \frac{1}{5}5x = x$

(d) $f(x) = 3\log_5(5x + 7)$, $g(x) = 5^{(x-3)/3} - 1.4$

$y = 3\log_5(5x+7)$
$y/3 = \log_5(5x+7)$
$5^{y/3} = 5x + 7$
$5^{y/3} - 7 = 5x$
$\frac{5^{y/3}}{5} - \frac{7}{5} = x$
$f^{-1}(x) = 5^{(x-3)/3} - 1.4$

or

$f(g(x)) = 3\log_5(5g(x) + 7)$
$= 3\log_5(5^{x/3} - 7 + 7)$
$= 3\log_5(5^{x/3})$
$= 3\,x/3$
$= x$

$g(f(x)) = 5^{\frac{(f(x) - 3)}{3}} - 1.4$
$= 5^{\log_5(5x+7) - 1} - 1.4$
$= \frac{1}{5}(5x+7) - 1.4$
$= x$

Cont.

3. In each of the following, find $f(g(x))$ and *simplify your answer.*

(a) $f(x) = \dfrac{1}{x-3}$, $g(x) = \dfrac{1+3x}{x}$

$$f(g(x)) = (g(x)-3)^{-1} = \left(\frac{1+3x}{x} - 3\right)^{-1} = \left(\frac{1+3x-3x}{x}\right)^{-1}$$

$$= \left(\frac{1}{x}\right)^{-1} = x$$

Answer: $f(g(x)) = x$

(b) $f(x) = 4x^2 + 4x + 1$, $g(x) = x + 3$

$$f(x) = (2x+1)^2$$

$$f(g(x)) = (2g(x)+1)^2 = (2(x+3)+1) = (2x+7)^2$$

Answer: $(2x+7)^2 = f(g(x))$

(c) $f(x) = 9x^2 + 6x + 1$, $g(x) = x - 1$

$$f(x) = (3x+1)^2$$

$$f(g(x)) = (3g(x)+1)^2 = (3(x-1)+1)^2$$

$$= (3x-2)^2$$

Answer: $(3x-2)^2 = f(g(x))$

4. Find $f(g(x))$ if $f(x) = b^2x^2 + 2bx + 1$ and $g(x) = x + a$. Simplify your answer.

$$f(x) = (bx+1)^2$$

$$f(g(x)) = (bg(x)+1)^2 = (b(x+a)+1)^2$$

$$= (bx + ab+1)^2$$

Answer: $f(g(x)) = (bx+(ab+1))^2$

5. Find the inverse of $f(x) = \dfrac{x^3 + 3ax^2 + 3a^2x + a^3}{(x+b)^3}$

$$y = \frac{x^3+3ax+3a^2x+a^3}{(x+b)^3} = \frac{(x+a)^3}{(x+b)^3}$$

$$y^{1/3} = \frac{x+a}{x+b} \implies (x+b)y^{1/3} = x+a$$

$$\implies xy^{1/3} + by^{1/3} = x+a \implies by^{1/3} - a = x - xy^{1/3}$$

$$\implies by^{1/3} - a = x(1-y^{1/3}) \implies \frac{by^{1/3}-a}{1-y^{1/3}} = x$$

Answer: $f^{-1}(x) = \dfrac{bx^{1/3}-a}{1-x^{1/3}}$

The End.