MATH 111*i* | HOMEWORK 5 | DUE THURSDAY, OCTOBER 12TH

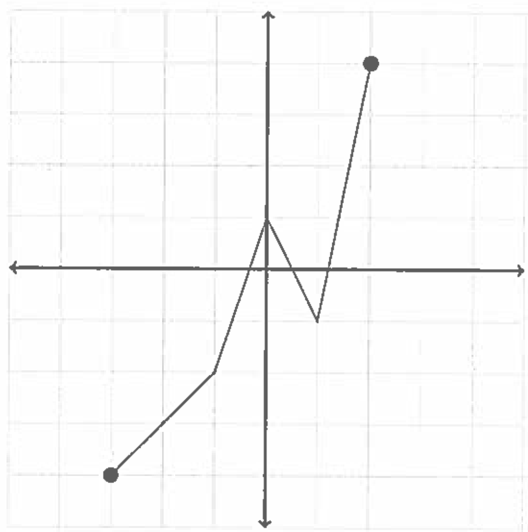
Name: Solutions

> This assignment is worth 100 points. You will be awarded 40 points for attempting the entire assignment (that is answer all problems). All problems will be graded for the remaining 60 points. The space left between each question is indicative of how much work you should show. If there are any problems you find particularly difficult, circle them in red. If there are any particular questions you would like feedback on, circle them in green. These are examples of questions that might appear on an exam/quiz. If you use a calculator to help, make sure you can also do them without it.

1. For each of the graphs below, determine each of the following. You may assume that each square is of length 1.

- The domain of the function
- The range of the function
- All, if any, local minimums
- All, if any, local maximums
- Global minimum, if it exists
- Global maximum, if it exists
- The interval(s) when the function is increasing
- The interval(s) when the function is decreasing

(a)



Domain: [-3, 2]

Range: [-4, 4]

Local minimums: (1, -1)

Local maximums: (0, 1)

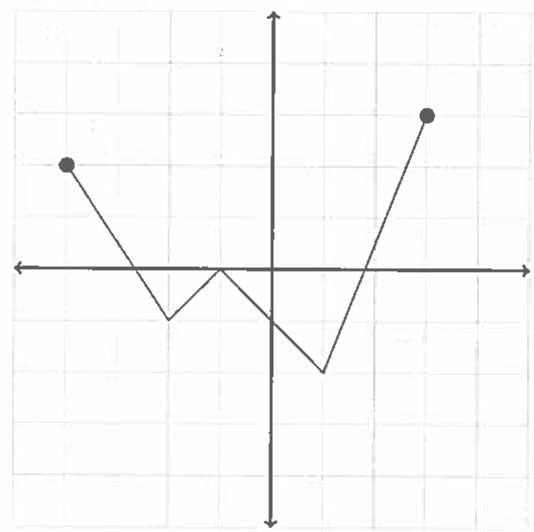
Global minimum: (-3, -4)

Global maximum: (2, 4)

Increasing: (-3, 0), (1, 2)

Decreasing: (0, 1)

(b)



Domain: [-4, 3]

Range: [-2, 3]

Local minimums: (-2, -1), (1, -2)

Local maximums: (-1, 0)

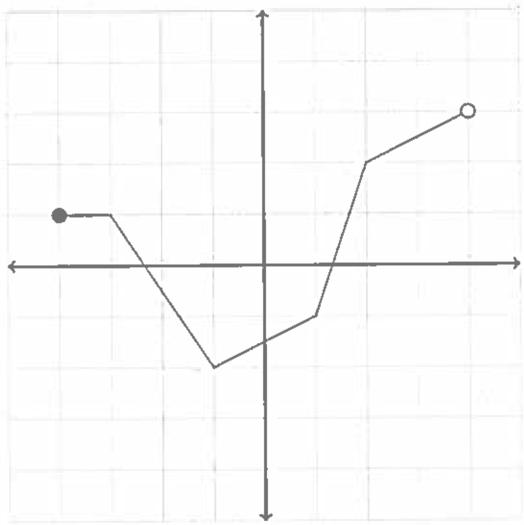
Global minimum: (1, -2)

Global maximum: (3, 3

Increasing: (-2, -1), (1, 3)

Decreasing: (-4, -2), (-1, 1)

(c)



Domain:
$[-4,4)$

Global minimum:
$(-1,-2)$

Range:
$[-2,3)$

Global maximum:
N/A

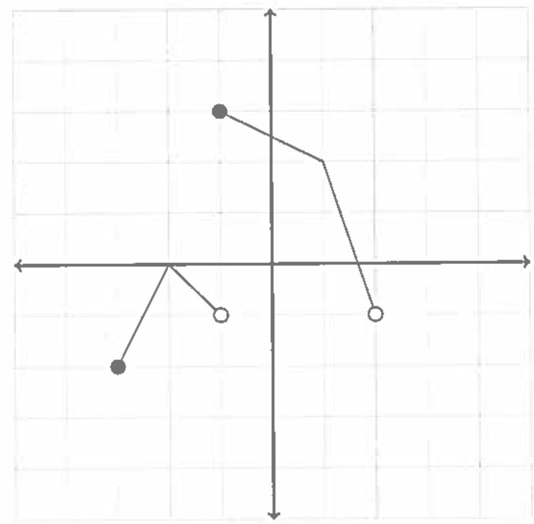
Local minimums:
$(-1,-2)$

Increasing:
$(-1,4)$

Local maximums:
any point in $(-4,-3)$

Decreasing:
$(-3,-1)$

(d)



Domain:
$[-3,2)$

Global minimum:
$(-3,-2)$

Range:
$[-2,3]$

Global maximum:
$(-1,3)$

Local minimums:
N/A

Increasing:
$(-3,-2)$

Local maximums:
$(-2,0)$

Decreasing:
$(-2,-1)$, $(-1,2)$

2. Consider the following modeling problems.

(a) A tee shirt company makes tee shirts with school logos. The company charges a fixed fee of \$200 to set up the machines plus \$3.50 per tee shirt.

i. Find a function C that models the cost of purchasing x tee shirts.

Answer: $C(x) = 200 + 3.5x$

ii. Use the model to find the cost of purchasing 600 tee shirts.

$200 + 3.5(600)$

Answer: \$2300

Cont.

(b) A flea market charges vendors a fixed fee of \$60 per month plus 75 cents per square foot for renting a space.

i. Find a function C that models the cost for one month's rental of a space with area x square feet.

Answer: $C(x) = 60 + 0.75x$

ii. Use the model to find the cost of one month's rental of a space with area 105 square feet.

$60 + 0.75(105)$

Answer: \$138.75

(c) The cost of driving a car depends on the number of miles driven and the gas mileage of the car. Kristi owns a Honda Accord that gets 30 miles to the gallon.

i. Find a function C that models the cost of driving Kristi's car x miles if the cost of gas is \$3.20 per gallon.

$$\text{cost} = \text{cost per mile} \times \#\text{ miles}$$
$$= \frac{\text{cost per gallon}}{\text{miles per gallon}} \times \#\text{miles}$$

Answer: $C(x) = \frac{3.2}{30}x = \frac{8}{75}x$

ii. Use the model to find the cost of driving Kristi's car 500 miles.

$$\frac{8}{75}(500) = \frac{8}{15}(100) = \frac{8}{3}(20)$$

Answer: $\$\frac{160}{3}$

iii. Kristi's budget for gas is \$250 per month. Use the model to find the number of miles Kristi can drive each month without exceeding her monthly gas budget.

$$\frac{8}{75}x = 250$$
$$x = \frac{250 \times 75}{8} = \frac{125 \times 75}{4} = \frac{5^3 \cdot 3 \cdot 5^2}{2^2} = \frac{3125 \cdot 3}{4}$$

Answer: $\frac{9375}{4}$ miles.

(d) For this question, you may want to use a calculator. Note that on an exam the numbers involved would be nice enough to do by hand.
Jason travels from his home in Connecticut to Germany to visit his grandparents. At the time, the euro/dollar exchange rate was €1 = $1.5532.

i. Find a function A that models the number of U.S. dollars required to purchase x euros.

$A(x) =$ cost per euro × # euros

Answer: $A(x) = 1.5532x$

ii. Jason bought a vase in Hamburg for his grandmother for €153. Use the model to find the price of the vase in U.S. dollars.

$A(x) = 1.5532(153)$

Answer: $237.64

iii. The day before returning home, Jason found he had 200 U.S. dollars worth of traveler's cheques left. He decided to convert these to euros to spend in Germany. Use the model to find how many euros he received for his $200.

$200 = 1.5532x$

$$x = \frac{200}{1.5532} = \frac{200}{\frac{15532}{10000}} = \frac{2000000}{15532} = \frac{500000}{3883}$$

Answer: €128.77

(e) A breakfast cereal manufacturer packages cereal in boxes that are 4 inches taller than they are wide and always have a depth of 3 inches.

i. Find a function V that models the volume of a cereal box that is x inches wide.

Answer: $V(x) = x(x+4)\cdot 3$

ii. Use the model to find the volume of a cereal box that is 10 inches wide.

$10(10+4)\cdot 3$

Answer: 420in^3

iii. The manufacturer makes a box of wheat bran cereal with a volume of 300in^3. Determine the width of the box.

$3x(x+4) = 300$

$x^2+4x = 100$

$(x+2)^2-4 = 100$

$(x+2)^2 = 104$

Answer: $x = \sqrt{104}-2$ in.

(f) A shipping company uses boxes that have square ends and are twice as long as they are wide.

i. Find a function S that models the surface area of a box whose square end is x inches.

$2x^2+4\cdot 2x^2$

Answer: $S(x)=10x^2$

ii. The company uses boxes that are 8 inches wide to ship cans of beans. Use the model to find the area of the material used to make each box.

$S(8)=10(8)^2$

Answer: $640 in^2$

iii. A box that ships a dozen cans of soup has a surface area of 330in^2. Determine how wide the box is.

$10x^2=330$

$x^2=33$

Answer: $x=\sqrt{33}$ in

iv. To ensure that the boxes are strong enough to safely hold their contents, they should have a surface area no larger than 550in^2. Find all possible widths of boxes the manufacturer can use.

$10x^2\leq 550$

$x^2\leq 55$

Answer: $x\leq\sqrt{55}$

The End.