

Solutions

Name: _____

This assignment is worth 100 points. You will be awarded 40 points for attempting the entire assignment (that is answer all problems). I will then select 20 problems at random to grade, each worth 3 points each. The space left between each question is indicative of how much work you should show. If there are any problems you find particularly difficult, circle them in red. If there are any particular questions you would like feedback on, circle them in green. These are examples of questions that might appear on an exam/quiz. If you use a calculator to help, make sure you can also do them without it.

1. Perform the operation and combine like terms for the following polynomials.

(a) $(10x^5 + 2x^3 - 1) + (8x^4 - x^3 + 16x^2)$

Answer: $10x^5 + 8x^4 + x^3 + 16x^2 - 1$

(b) $(7t^2 - 13t + 4) + (-6t^2 + 13t - 4)$

Answer: t^2

(c) $(z^3 + 2z^2 - 15z + 7) - (-12z^2 + 9z - 3)$

Answer: $z^3 + 14z^2 - 24z + 10$

(d) $(150x^3 + 8x - 14) - (100x^4 - 19x^2 - 7x)$

Answer: $-100x^4 + 150x^3 + 19x^2 + 15x - 14$

(e) $w^5 - (w^4 + w^3 + w^2 + w + 1)$

Answer: $w^5 - w^4 - w^3 - w^2 - w - 1$

(f) $6y^2(3 - y^2 + 2y^3)$

Answer: $12y^5 - 6y^4 + 18y^2$

(g) $x^9(x^2 + 7x - 4)$

Answer: $x^{11} + 7x^{10} - 4x^9$

(h) $(7x - 5)(4 - 10x)$

$$= 7x \cdot 4 - 7x \cdot 10x - 5 \cdot 4 + 5 \cdot 10x$$

$$= 28x - 70x^2 - 20 + 50x$$

Answer: $-70x^2 + 78x - 20$

(i) $(4 + 9t^2)(t^3 - 3t)$

$$= 4 \cdot t^3 - 4 \cdot 3t + 9t^2 \cdot t^3 - 9t^2 \cdot 3t$$

$$= 4t^3 - 12t + 9t^5 - 27t^3$$

Answer: $9t^5 - 23t^3 - 12t$

(j) $(1 + 8y)(y^3 - 4y^2 + 7)$

Answer: $8y^4 - 31y^3 - 4y^2 + 56y + 7$

(k) $7(x - 9)(2x + 3)$

Answer: $14x^2 - 105x - 189$

(l) $z^2(1 - z^2)(1 + z^2)$

Answer: $-z^6 + z^2$

(m) $(2 - x + 4x^2)(6x + 7)$

Answer: $24x^3 + 22x^2 + 5x + 14$

(n) $(10w^2 - 4w + 9)(w^3 + 5w^2 + 2)$

Answer: $10w^5 + 46w^4 - 11w^3 + 65w^2 - 8w + 18$

(o) $10(x + 3x^2)^2$

Answer: $90x^4 + 60x^3 + 10x^2$

(p) $(1 - 5y)(4 + y)^2$

Answer: $-5y^3 - 39y^2 - 72y + 16$

(q) $(x^2 - 7x + 10) - (3 - x)(3 + x)$

Answer: $2x^2 - 7x + 1$

(r) $(x + 9x^3)^2 - (4x^2 - 1)^2$

$$= x^2 + 18x^4 + 81x^6 - 16x^4 + 8x^2 - 1$$

Answer: $81x^6 + 2x^4 + 9x^2 - 1$

2. Think about the following statements and carefully explain your answer.

(a) If we multiply a polynomial with degree m and a polynomial of degree n , what is the degree of the resulting polynomial?

$$\begin{array}{ccccccc} (1 + x^2)(1 + x^3) & = & 1 + x^2 + x^3 + x^5 \\ \uparrow & & \uparrow & & \uparrow & & \\ \text{deg. } 2 & & \text{deg. } 3 & & \text{deg. } 5 = 2 + 3 & & \end{array}$$

$$\boxed{n + m}$$

(b) If we add a polynomial of degree m to a polynomial of degree $n < m$, what is the degree of the resulting polynomial?

$$(1+x^2) + (1+x^3) = 2+x^2+x^3$$

\uparrow \uparrow \uparrow
 deg. 2 deg. 3 deg. 3

M

(c) If we add 2 polynomials of the same degree, say n , can the resulting polynomial have a degree different to n ? Give an example if this is possible.

$$(1+x^2) + (1-x^2) = 2$$

\uparrow \uparrow \uparrow
 deg. 2 deg. 2 deg. 0

Yes

3. Factor out the greatest common factor from each polynomial.

(a) $x^3 - 6x^8 + 10x^{10}$

Answer: $x^3(1 - 6x^5 + 10x^7)$

(b) $25u^6 - 15u^5 + 30u^8$

Answer: $5u^5(5u - 3 + 6u^3)$

(c) $2y^6z - y^4z^{10} + 3y^2z^2$

Answer: $y^2z(2y^4 - y^2z^9 + 3z)$

(d) $7a^{10}b^7 + 14a^8b^9 - 35a^6b^{12}$

Answer: $7a^6b^7(a^4 + 2a^2b^2 - 5b^5)$

(e) $3(9 + 7x) - (2 - x)(9 + 7x)$

Answer: $(9 + 7x)(x + 1)$

$$\begin{aligned}
 \text{(f)} \quad & z^2(4z - z^3) + 7(z^3 - 4z) \\
 & = z^3(4 - z^2) - 7z(4 - z^2) \\
 & = z(z^2 - 7)(4 - z^2)
 \end{aligned}$$

Answer: $z(z^2 - 7)(2 - z)(2 + z)$

$$\text{(g)} \quad 8y(2y + 7)^4 - 2y^3(2y + 7)^9$$

Answer: $2y(2y + 7)^4(4 - y^2(2y + 7)^5)$

$$\text{(h)} \quad w^2(1 + w^2)(8w - 1)^{10} + 9w(1 + w^2)^4(8w - 1)^7$$

Answer: $w(8w - 1)^7(1 + w^2)(w(8w - 1)^3 + 9(1 + w^2)^3)$

4. Factor each of the following by grouping.

$$\text{(a)} \quad 18x - 2x^3 + 9 - x^2$$

$$\begin{aligned}
 & = 2x(9 - x^2) + 1 \cdot (9 - x^2) \\
 & = (2x + 1)(9 - x^2)
 \end{aligned}$$

Answer: $(2x + 1)(3 - x)(3 + x)$

$$\text{(b)} \quad 6w^4 + 3w^3 - 14w^2 - 7w$$

$$\begin{aligned}
 & = 3w^3(2w + 1) - 7w(2w + 1) \\
 & = (3w^3 - 7w)(2w + 1)
 \end{aligned}$$

Answer: $w(3w^2 - 7)(2w + 1)$

$$\text{(c)} \quad y^4 + y^3 + 9y^2 + 9y$$

$$\begin{aligned}
 & = y^3(y + 1) + 9y^2(y + 1) \\
 & = (y^3 + 9y^2)(y + 1)
 \end{aligned}$$

Answer: $y^2(y + 9)(y + 1)$

$$\text{(d)} \quad 21x - 56x^4 - 12x^3 + 32x^6$$

$$\begin{aligned}
 & = 7x(3 - 8x^3) - 4x^3(3 - 8x^3) \\
 & = (7x - 4x^3)(3 - 8x^3)
 \end{aligned}$$

Answer: $x(7 - 4x^2)(3 - 8x^3)$

$$\text{(e)} \quad 6t^3 + 3t^4 - 2t^5 - t^6$$

$$\begin{aligned}
 & = 3t^3(2 + t) - t^5(2 + t) \\
 & = (3t^3 - t^5)(2 + t)
 \end{aligned}$$

Answer: $t^3(3 - t^2)(2 + t)$

5. Factor each of the following.

(a) $x^2 - 10x + 9$

Answer: $(x-1)(x-9)$

(b) $t^2 + 11t + 24$

Answer: $(t+3)(t+8)$

(c) $z^2 - 9z - 10$

Answer: $(z-10)(z+1)$

(d) $x^2 - 3x - 28$

Answer: $(x-7)(x+4)$

(e) $x^2 + 10x - 24$

Answer: $(x+12)(x-2)$

(f) $w^2 - 8w + 16$

Answer: $(w-4)^2$

(g) $z^2 + 6z + 9$

Answer: $(z+3)^2$

(h) $x^2 - 144$

Answer: $(x-12)(x+12)$

(i) $36 - x^2$

Answer: $(6-x)(6+x)$

(j) $4z^2 - 23z - 6$

Answer: $(4z+1)(z-6)$

(k) $2y^2 - 9y + 10$

Answer: $(2y-5)(y-2)$

(l) $12x^2 + 31x + 7$

Answer: $(4x+1)(3x+7)$

(m) $6z^2 - 35z + 36$

Answer: $(2z-9)(3z-4)$

(n) $8t^2 + 29t - 12$

Answer: $(8t-3)(t+4)$

(o) $21 - w - 2w^2$

Answer: $-(2w+7)(w-3)$

(p) $36v^2 - 49$

Answer: $(6v-7)(6v+7)$

(q) $100x^2 + 20x + 1$

Answer: $(10x+1)^2$

(r) $25z^2 - 40z + 16$

Answer: $(5z-4)^2$

(s) $9y^2 - 121$

Answer: $(3y-11)(3y+11)$

6. Factor each of the following

(a) $4x^3 - 20x^2 - 144x$

Answer: $4x(x-9)(x+4)$

(b) $t^4 + 15t^3 + 14t^2$

Answer: $t^2(t+1)(t+14)$

(c) $6u^8 - 3u^6 - 3u^4$

Answer: $3u^4(u-1)(u+1)(2u^2+1)$

(d) $t^8 + 5t^4 - 24$

Answer: $(t^4-3)(t^4+8)$

(e) $2z^4 - 5z^2 - 12$

Answer: $(z-2)(z+2)(2z^2+3)$

(f) $4x^6 + x^3 - 5$

Answer: $(x-1)(x^2+x+1)(4x^3+5)$