Solutions

Name:_____

> This assignment is worth 100 points. You will be awarded 40 points for attempting the entire assign-
> ment (that is answer all problems). I will then select 30 problems at random to grade, each worth 2
> points each. The space left between each question is indicative of how much work you should show.
> If there are any problems you find particularly difficult. circle them in red. If there are any particular
> questions you would like feedback on, circle them in green. These are examples of questions that
> might appear on an exam/quiz. If you use a calculator to help, make sure you can also do them
> without it.

1. Given the sets below, find the indicated set. Remember $\{a, b\}$ denotes the set containing a and b, and (a, b) denotes the interval from a to b.

   (a) $A = \{1, 2, 5, 7, 8, 9\}$, $A \cup B =$

        $B = \{2, 3, 4, 8, 9, 10\}$.

                          Answer: $\{1, 2, 3, 4, 5, 7, 8, 9, 10\}$

   (b) $S = \{2, 4, 6, 8, 10, 12\}$, $S \cap T \cap U =$

        $T = \{3, 6, 9, 12, 15\}$,

        $U = \{2, 3, 6, 8, 9, 12\}$.

                          Answer: $\{6, 12\}$

   (c) $X = \{1, 2, 3, 5, 8, 13\}$, $X \cap Y =$

        $Y = (2, 9)$.

                          Answer: $\{3, 5, 8\}$

   (d) $M = (-2, 6]$, $M \cup N =$

        $N = [-1, 6]$.

                          Answer: $(-2, 6]$

2. For each of the sets A and B, find a point that lies in one set. but not the other. State which set the point you choose belongs to.

   (a) $A = (2, 5)$, $B = [3, 4]$.

                          Answer: 4.5 , A

   (b) $A = [-7, 3)$, $B = (-7, 3]$.

                          Answer: -7 , A

   (c) $A = [0, 17)$, $B = (-1, 15)$.

                          Answer: 16 , A

   (d) $A = (8, 10]$, $B = (8, 9.9]$.

                          Answer: 9.95 , A

3. Calculate the given expression. Your answer should be a single number with no exponents or radicals.

(a) $64^{\frac{1}{2}}$ $= \sqrt{64}$

Answer: $\underline{\quad 8 \quad}$

(b) $16^{\frac{1}{2}}$ $= \sqrt{16}$

Answer: $\underline{\quad 4 \quad}$

(c) $16^{\frac{1}{4}}$ $= \sqrt[4]{16}$
$= \sqrt{4}$

Answer: $\underline{\quad 2 \quad}$

(d) $121^{-\frac{1}{2}}$
$= \dfrac{1}{\sqrt{121}}$

Answer: $\underline{\quad \frac{1}{11} \quad}$

(e) $(-64)^{-\frac{1}{3}}$
$= \dfrac{1}{\sqrt[3]{-64}}$

Answer: $\underline{\quad -4 \quad}$

(f) $\left(-\frac{27}{8}\right)^{\frac{1}{3}}$
$= \dfrac{\sqrt[3]{-27}}{\sqrt[3]{8}}$

Answer: $\underline{\quad -\frac{3}{2} \quad}$

(g) $49^{\frac{3}{2}}$
$= \left(\sqrt{49}\right)^3$
$= 7^3$

Answer: $\underline{\quad 343 \quad}$

(h) $64^{-\frac{5}{6}}$
$= \left(\dfrac{1}{\sqrt[6]{64}}\right)^5 = \left(\dfrac{1}{2}\right)^5$

Answer: $\underline{\quad \frac{1}{32} \quad}$

(i) $(-243)^{\frac{1}{5}}$ $= \sqrt[5]{-243}$

Answer: $\underline{\quad -3 \quad}$

(j) $\sqrt{256}$

Answer: $\underline{\quad 16 \quad}$

(k) $\sqrt[4]{256}$ $= \sqrt{16}$

Answer: $\underline{\quad 4 \quad}$

(l) $\sqrt[8]{256} = \sqrt[4]{16}$
$= \sqrt{4}$

Answer: $\underline{\quad 2 \quad}$

(m) $\sqrt[5]{-1024}$

Answer: $\underline{\quad -4 \quad}$

(n) $\sqrt[3]{-216}$

Answer: $\underline{\quad -6 \quad}$

(o) $\sqrt[3]{343}$

Answer: $\underline{\quad 7 \quad}$

4. Simplify the given expression so that the only exponents that appear are positive.

(a) $(3x^{-2}y^{-4})^{-1}$

$= 3^{-1}x^2 y^4$

Answer: $\underline{\quad \dfrac{x^2 y^4}{3} \quad}$

(b) $\left[(2a^2)^{-3}b^4\right]^{-3}$

$= \left[2^{-3}a^{-6}b^4\right]^{-3}$
$= 2^9 a^{18} b^{-12}$

Answer: $\underline{\quad \dfrac{2^9 a^{18}}{b^{12}} \quad}$

(c) $\dfrac{c^{-6}b^{10}}{b^9 c^{-11}}$

$= \dfrac{b^{10} c^{11}}{b^9 c^6}$

Answer: $\underline{\quad bc^5 \quad}$

(d) $\dfrac{4a^3\,(b^2a)^{-4}}{c^{-6}a^2b^{-7}}$ $= \dfrac{4a^3\,b^{-8}a^{-4}}{c^{-6}a^2b^{-7}} = \dfrac{4a^3b^7c^6}{a^4a^2b^8} = \dfrac{4a^3b^7c^6}{a^6b^8}$

Answer: $\dfrac{4c^6}{a^3b}$

(e) $\dfrac{(6v^2)^{-1}\,x^{-4}}{(2v)^{-3}\,w^{10}}$ $= \dfrac{6^{-1}v^{-2}x^{-4}}{2^{-3}v^{-3}w^{10}} = \dfrac{2^3\,v^3}{6\,v^2w^{10}x^4}$

Answer: $\dfrac{4v}{3w^{10}x^4}$

(f) $\left[\dfrac{(8x^{21})^0\,y^{-3}x^8}{y^{-9}x^{-1}}\right]^6$ $= \left[\dfrac{1\cdot x^8\cdot x\cdot y^9}{y^3}\right]^6 = \left[x^9y^6\right]^6$

Answer: $x^{54}y^{36}$

(g) $\left(\dfrac{a^2b^{-4}c^{-1}}{b^{-9}c^8a^{-4}}\right)^{-2}$ $= \left(\dfrac{a^2a^4b^9}{b^4c\cdot c^8}\right)^{-2} = \left(\dfrac{b^4c^9}{a^6b^9}\right)^2 = \left(\dfrac{c^9}{a^6b^5}\right)^2$

Answer: $\dfrac{c^{18}}{a^{12}b^{10}}$

(h) $\left[\dfrac{p^{-6}q^7\,(p^2q)^{-3}}{(p^{-1}q^{-4})^2\,p^{10}}\right]$ $= \dfrac{p^{-6}q^7\,p^{-6}q^{-3}}{p^{-2}q^{-8}\,p^{10}} = \dfrac{p^{-12}q^4}{p^8q^{-8}} = \dfrac{q^4q^8}{p^8p^{12}}$

Answer: $\dfrac{q^{12}}{p^{20}}$

5. Simplify the given expression so that the only exponents that appear are positive.

(a) $(p^{-2}q^{-4})^{\frac{3}{2}}$ $= p^{-3}q^{-6}$

Answer: $\dfrac{1}{p^3q^6}$

(b) $x^{\frac{3}{4}}\left(x^2x^{-\frac{1}{3}}\right)^{\frac{3}{2}}$ $= x^{3/4}\cdot x^3\cdot x^{-1/2}$
$= x^{3/4}\cdot x^{12/4}\cdot x^{-2/4}$

Answer: $x^{13/4}$

(c) $a^{\frac{1}{2}}a^{-\frac{1}{3}}a^{\frac{1}{4}}$
$= a^{6/12}\cdot a^{-4/12}\cdot a^{3/12}$

Answer: $a^{5/12}$

Cont.

(d) $\left(m^{-\frac{7}{3}}n^{\frac{5}{4}}\right)^{-\frac{8}{6}}$ $= m^{56/27}\, n^{-10/9}$

Answer: $\dfrac{m^{56/27}}{n^{10/9}}$

(e) $\left(\dfrac{a^{-\frac{1}{3}}b^2}{b^{\frac{2}{3}}a^{-\frac{1}{4}}}\right)^{\frac{1}{5}}$ $= \left(\dfrac{a^{3/4}\,b^2}{a^{1/3}\,b^{2/3}}\right)^{1/5} = \left(\dfrac{a^{9/12}\,b^{6/3}}{a^{4/12}\,b^{2/3}}\right)^{1/5} = \left(a^{5/12}\,b^{4/3}\right)^{1/5}$

$a^{1/12}\,b^{4/15}$

Answer: $a^{1/12}\,b^{4/15}$

(f) $\left(\dfrac{p^{\frac{1}{2}}q^{\frac{1}{3}}}{p^{-\frac{1}{3}}q^{-\frac{1}{4}}}\right)^{-3}$ $= \left(p^{1/2}p^{1/3}q^{1/3}q^{1/4}\right)^{-3}$
$= \left(p^{5/6}\,q^{7/12}\right)^{-3}$

Answer: $\dfrac{1}{p^{5/2}q^{7/4}}$

(g) $\left(\dfrac{x^{\frac{3}{4}}y^{-\frac{2}{3}}}{x^{\frac{7}{4}}}\right)^{\frac{7}{8}}$ $= \left(\dfrac{x^{3/4}}{x^{7/4}\,y^{2/3}}\right)^{7/8} = \left(\dfrac{1}{x\,y^{2/3}}\right)^{7/8}$

Answer: $\dfrac{1}{x^{7/8}\cdot y^{7/12}}$

(h) $\left(\dfrac{b^3c^{-\frac{1}{4}}a^{-1}}{b^{\frac{1}{4}}a^{-\frac{2}{7}}c^{\frac{3}{2}}}\right)^{\frac{2}{3}}$ $= \left(\dfrac{a^{2/7}b^3}{a\cdot b^{1/4}c^{3/2}c^{1/4}}\right)^{2/3} = \left(\dfrac{a^{2/7}\,b^{12/4}}{a^{7/7}\,b^{1/4}\,c^{7/4}}\right)^{2/3} = \left(\dfrac{b^{11/4}}{a^{5/7}\,c^{7/4}}\right)^{2/3}$

Answer: $\dfrac{b^{11/6}}{a^{10/21}c^{7/6}}$

6. Write the given expression in exponential form.

(a) $\sqrt{3n}$

Answer: $3^{1/2}\,n^{1/2}$

(b) $\sqrt[6]{2y}$

Answer: $2^{1/6}\cdot y^{1/6}$

(c) $\sqrt[5]{7x^3}$

Answer: $7^{1/5}\cdot x^{3/5}$

Cont.

(d) $\sqrt[4]{xyz}$

Answer: $x^{1/4} \cdot y^{1/4} \cdot z^{1/4}$

(e) $\sqrt{x+y}$

Answer: $(x+y)^{1/2}$

(f) $\sqrt[3]{a^3 + b^3}$

Answer: $(a^3 + b^3)^{1/3}$

7. Convert the given expression to rational exponent form.

(a) $\sqrt{z^5}$

Answer: $z^{5/2}$

(b) $\sqrt[3]{z^5}$

Answer: $z^{5/3}$

(c) $\sqrt[3]{16x^{17}}$

Answer: $16^{1/3} \cdot x^{17/3}$

(d) $\sqrt[6]{128y^{11}}$

Answer: $128^{1/6} \cdot y^{11/6}$

(e) $\sqrt{x^3 y^{17} z^4}$

Answer: $x^{3/2} y^{17/2} z^2$

(f) $\sqrt[4]{x^3 y^{20} z^5}$

Answer: $x^{3/4} y^5 z^{5/4}$

(g) $\sqrt[4]{729x^7yz^{13}}$

Answer: $729^{1/4} \cdot x^{7/4} \cdot y^{1/4} \cdot z^{13/4}$

(h) $\sqrt[3]{4x^2y}\sqrt[3]{10x^5y^2}$ $= \left(4 \cdot x^2 \cdot y \cdot 10 \cdot x^5 \cdot y^2\right)^{1/3} = \left(40 \cdot x^7 \cdot y^3\right)^{1/3}$

Answer: $2 \cdot 5^{1/3} \cdot x^{7/3} \cdot y$

(i) $\sqrt{3x}\sqrt{6x}\sqrt{14x}$ $= \left(3x \cdot 6x \cdot 14x\right)^{1/2} = \left(6^2 \cdot 7 x^3\right)^{1/2}$

Answer: $6 \cdot 7^{1/2} x^{3/2}$

(j) $\sqrt[4]{2xy^3}\sqrt[4]{32x^2y^2}$ $= \left(2xy^3 \cdot 32x^2y^2\right)^{1/4} = \left(4 \cdot 16 x^3 y^5\right)^{1/4}$

Answer: $2^{3/2} \cdot x^{3/4} \cdot y^{5/4}$

8. Decide if the given statement is true or false. If false give the corrected version of the statement.

(a) $\frac{1}{6x} = 6x^{-1}$ $6x^{-1} = \frac{6}{x} \neq \frac{1}{6x}$

Answer: False

(b) $\left(x^3\right)^7 = x^{10}$

$\left(x^3\right)^7 = x^{21} \neq x^{10}$

Answer: False

(c) $\left(m^3n^4\right)^2 = m^{12}n^8$

$\left(m^3n^4\right)^2 = m^6 n^8 \neq m^{12}n^8$

Answer: False

(d) $\left[\left(z^2\right)^3\right]^4 = z^{24}$

$= \left(z^6\right)^4 = z^{24}$

Answer: True.

(e) $(x+y)^3 = x^3 + y^3$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \neq x^3 + y^3$$

Answer: _____ False _____

(f) $a^{-\frac{3}{2}} = a^{\frac{2}{3}}$

$$a^{-3/2} = (a^{3/2})^{-1} = \frac{1}{a^{3/2}} \neq a^{2/3}$$

Answer: _____ False _____

(g) $x^{-n} = x^{\frac{1}{n}}$

$$x^{-n} = (x^n)^{-1} = \frac{1}{x^n} \neq x^{1/n}$$

Answer: _____ False _____

(h) $3x^{\frac{1}{2}} = \sqrt{3x}$

$$3x^{1/2} = 3\sqrt{x} \neq \sqrt{3x}$$

Answer: _____ False _____

(i) $\sqrt[3]{x+6} = \sqrt[3]{x} + \sqrt[3]{6}$

$$\left[(x+6)^{1/3}\right]^3 = x+6$$
$$(\sqrt[3]{x} + \sqrt[3]{6})^3 = x + 3\sqrt[3]{6x^2} + 3\sqrt[3]{36x} + 6$$

Answer: _____ False _____

(j) $\sqrt[4]{x^2} = \sqrt{x}$

$$\sqrt[4]{x^2} = (x^2)^{1/4} = x^{1/2} = \sqrt{x}$$

Answer: _____ True _____

9. Expand and simplify the given expression.

(a) $3(3w + 3x) + 3(2w - 2x)$

$$3 \cdot 3w + 3 \cdot 3x + 3 \cdot 2w - 3 \cdot 2x$$
$$= 9w + 9x + 6w - 6x$$

Answer: _____ $15w + 3x$ _____

(b) $4(6d + 2e) - 6(3d + 2e)$

$$= 4 \cdot 6d + 4 \cdot 2e - 6 \cdot 3d - 6 \cdot 2e$$
$$= 24d + 8e - 18d - 12e$$

Answer: $6d - 4e$

(c) $x(x + 2y - 3z) + 3z(a - 2y + 3z) - 2y(x - y - 3z)$

$$= x \cdot x + x \cdot 2y - x \cdot 3z + 3z \cdot a - 3z \cdot 2y + 3z \cdot 3z - 2y \cdot x + 2y \cdot y + 2y \cdot 3z$$
$$= x^2 + 2xy - 3xz + 3az - 6yz + 9z^2 - 2xy + 2y^2 + 6yz$$

Answer: $x^2 - 3xz + 3az - 6yz + 9z^2 + 2y^2 + 6yz$

(d) $3a(2b - 3c + 4d) - 2b(3a - c + 6d)$

$$= 3a \cdot 2b - 3a \cdot 3c + 3a \cdot 4d \quad -2b \cdot 3a + 2b \cdot c - 2b \cdot 6d$$
$$= 6ab - 9ac + 12ad - 6ab + 2bc - 12bd$$

Answer: $-9ac + 12ad + 2bc - 12bd$

(e) $7(p + 2) - 3(2p - 3)$

$$7 \cdot p + 7 \cdot 2 - 3 \cdot 2p + 3 \cdot 3$$
$$= 7p + 14 - 6p + 9$$

Answer: $P + 23$

(f) $6f(f + 2g - h)(h + 3g)$

$$(6f \cdot f + 6f \cdot 2g - 6f \cdot h)(h + 3g) = 6f \cdot f \cdot h + 6f \cdot f \cdot 3g + 6f \cdot 2g \cdot h + 6f \cdot 2g \cdot 3g$$
$$- 6f \cdot h \cdot h - 6f \cdot h \cdot 3g$$
$$= 6f^2 h + 18f^2 g + 12fgh + 36fg^2 - 6fh^2 - 18fgh$$

Answer: $6f^2 h + 18f^2 g - 6fgh + 36fg^2 - 6fh^2$

(g) $5x + 6xy(z + 5y) + 5z(2x - 3y)(2x + 3y)$

$$= 5x + 6xy \cdot z + 6xy \cdot 5y + 5z(4x^2 - 9y^2)$$
$$= 5x + 6xyz + 30xy^2 + 5z \cdot 4x^2 - 5z \cdot 9y^2$$

Answer: $5x + 6xyz + 30xy^2 + 20x^2 z - 45y^2 z$

10. Factorise the given expression completely.

(a) $36b^2 - 25a^2$ $= (6b)^2 - (5a)^2$

Answer: $(6b - 5a)(6b + 5a)$

(b) $2xy - 6x^2y + 4xy^3$

Answer: $2xy(1 - 3x + 2y^2)$

(c) $d^2 + 4e^2 - 4de$

$= d^2 - 2d(2e) + (2e)^2$

Answer: $(d - 2e)^2$

(d) $u^2v^3 - 3uv + 7u^2v$

Answer: $uv(uv^2 - 3 + 7u)$

(e) $a^2 + 2ab + b^2$

Answer: $(a + b)^2$

(f) $xy(x + y)^2 + 4xy + 6x^2y$

$xy((x + y)^2 + 4 + 6x)$

Answer: $xy(x^2 + 6x + 2xy + y^2 + 4)$

The End.