

# EXTREMAL APPROXIMATELY CONVEX FUNCTIONS AND THE BEST CONSTANTS IN A THEOREM OF HYERS AND ULAM

S. J. DILWORTH, RALPH HOWARD, AND JAMES W. ROBERTS

ABSTRACT. Let  $n \geq 1$  and  $B \geq 2$ . A real-valued function  $f$  defined on the  $n$ -simplex  $\Delta_n$  is approximately convex with respect to  $\Delta_{B-1}$  if

$$f\left(\sum_{i=1}^B t_i x_i\right) \leq \sum_{i=1}^B t_i f(x_i) + 1$$

for all  $x_1, \dots, x_B \in \Delta_n$  and all  $(t_1, \dots, t_B) \in \Delta_{B-1}$ . We determine the extremal function of this type which vanishes on the vertices of  $\Delta_n$ . We also prove a stability theorem of Hyers-Ulam type which yields as a special case the best constants in the Hyers-Ulam stability theorem for  $\varepsilon$ -convex functions.

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