EXTREMAL APPROXIMATELY CONVEX FUNCTIONS AND THE BEST CONSTANTS IN A THEOREM OF HYERS AND ULAM

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ABSTRACT. Let $n \geq 1$ and $B \geq 2$. A real-valued function f defined on the n-simplex Δ_n is approximately convex with respect to Δ_{B-1} if

$$f\left(\sum_{i=1}^{B} t_i x_i\right) \leq \sum_{i=1}^{B} t_i f(x_i) + 1$$

for all $x_1, \ldots, x_B \in \Delta_n$ and all $(t_1, \ldots, t_B) \in \Delta_{B-1}$. We determine the extremal function of this type which vanishes on the vertices of Δ_n. We also prove a stability theorem of Hyers-Ulam type which yields as a special case the best constants in the Hyers-Ulam stability theorem for ε-convex functions.