

**A STRONG MAXIMUM PRINCIPLE FOR WEAK
SOLUTIONS OF QUASI-LINEAR ELLIPTIC EQUATIONS
WITH APPLICATIONS TO LORENTZIAN AND
RIEMANNIAN GEOMETRY**

LARS ANDERSSON, GREGORY J. GALLOWAY, AND RALPH HOWARD

ABSTRACT. The strong maximum principle is proved to hold for weak (in the sense of support functions) sub- and super-solutions to a class of quasi-linear elliptic equations that includes the mean curvature equation for C^0 spacelike hypersurfaces in a Lorentzian manifold. As one application a Lorentzian warped product splitting theorem is given.

CONTENTS

1. Introduction	2
2. The Analytic Maximum Principle	3
2.1. Outline of the Proof	5
2.2. Reduction to a Standard Setup	7
2.3. Some Calculus Lemmata	8
2.4. Proof of the Maximum Principle	13
3. Geometric Maximum Principles for Hypersurfaces in Lorentzian and Riemannian Manifolds	16
3.1. Reduction to the Analytic Maximum Principle	20
3.2. Proof of Theorem 3.6	22
3.3. A Geometric Maximum Principle for Riemannian Manifolds	24
4. Applications to Warped Product Splitting Theorems	25
4.1. Statement of Results	25
4.2. Proofs	28
References	36

1991 *Mathematics Subject Classification*. Primary: 58G03, 35B50 Secondary: 53C21, 83C75.

Key words and phrases. maximum principle, elliptic PDE, quasilinear PDE, mean curvature, splitting theorems, Lorentzian geometry.