Mathematics 546 Test #2 Name:______________________________

Show your work! Answers that do not have a justification will receive no credit.

(1) (25 Points) Define the following:

(a) The ***order*** of an element a of a group G.

(b) The ***cyclic subgroup*** $\langle a \rangle$ generated by an element a of a group G.

(c) H is a ***subgroup*** of the group G.

(d) If H is a subgroup of G and $x \in G$ then define the ***left coset*** xH.

(e) The groups G_1 and G_2 are ***isomorphic***.

(2) (10 Points) A partial group table is give given below. Complete the table.

	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	b	e	d		
b	b					
c	c	f		e		a
d	d				e	
f	f					

(3) (10 Points) Let G be the group $\mathbf{Z}_8^*$.

(a) Find the order of all the elements in $\mathbf{Z}_8^*$.

(b) Is $\mathbf{Z}_8^*$ cyclic? Justify your answer.

(4) (10 Points) In the group $\mathbf{Z}_3 \times \mathbf{Z}_3$ find all the cosets of the cyclic subgroup $H = \langle (1,2) \rangle$.

(5) (10 Points) Let H be a finite subset of a group that is closed under the group operations. Show that H contains the identity element.

(6) (10 Points) Let G be a group and $g_0 \in G$. Let $H = \{x \in G : xg_0 = g_0x\}$. (That is H is the set of elements of G that commute with g_0). Show that H is a subgroup of G.

(7) (10 Points) Prove that if H and K are subgroups of a group G, then $H \cap K$ is also a subgroup of G.

(8) (10 Points)

(a) Explain why $\mathbf{Z}_2$ and $\mathbf{Z}_3^*$ are isomorphic.

(b) Explain why $\mathbf{Z}_4$ and $\mathbf{Z}_8^*$ are not isomorphic.

(9) (10 Points) Let G be a group such that $x^2 = e$ for all $x \in G$. Then show that G is Abelian.