

**Mathematics 546 Test #1** Name: \_\_\_\_\_

**Show your work!** Answers that do not have a justification will receive no credit.

(1) (25 Points) Define the following:

(a)  $a$  is a ***factor*** of  $b$ .

(b)  $p$  is a ***prime***.

(c)  $d$  is the ***greatest common divisor*** of  $a$  and  $b$ .

(d) The subset  $I$  of  $\mathbf{Z}$  is an ***ideal***.

(e)  $(G, \cdot)$  is a ***group***.

(2) (20 Points) State the following:

(a) The principle of induction.

(b) The Trichotomy principle.

(c) The division algorithm.

(d) The “gcd= linear combo” theorem.

(3) (10 Points) Find a solution to  $4x \equiv 5 \pmod{13}$ .

(4) (10 Points) Find the greatest common divisor of 153 and 180 and express  $\gcd(153, 180)$  as a linear combination of 153 and 180.

(5) (10 Points) Prove that if  $a \mid b$  and  $b \mid c$ , then  $a \mid c$ .

(6) (10 Points) Give the addition and multiplication tables for arithmetic modulo 5.

(7) (10 Points) Prove that if  $a \equiv b \pmod{m}$ , then for any integer  $s$  that  $as \equiv bs \pmod{m}$ .

(8) (10 Points) Show that for any integer  $n$  that  $\gcd(n, n + 3)$  is either 1 or 3.