

Mathematics 546 Test #1

Name: _____
Show your work! Answers that do not have a justification will receive no credit.

(1) (25 Points) Define the following:

(a) a is a ***factor*** of b .

(b) p is a ***prime***.

(c) d is the ***greatest common divisor*** of a and b .

(d) The the subset I of \mathbf{Z} is an ***ideal***.

(e) (G, \cdot) is a ***group***.

(2) (20 Points) State the following:

(a) The principle of induction.

(b) The Trichotomy principle.

(c) The division algorithm.

(d) The “gcd= linear combo” theorem.

(3) (10 Points) Find a solution to $4x \equiv 5 \pmod{13}$.

(4) (10 Points) Find the greatest common divisor of 153 and 180 and express $\gcd(153, 180)$ as a linear combination of 153 and 180.

(5) (10 Points) Prove that if $a \mid b$ and $b \mid c$, then $a \mid c$.

(6) (10 Points) Give the addition and multiplication tables for arithmetic modulo 5.

(7) (10 Points) Prove that if $a \equiv b \pmod{m}$, then for any integer s that $as \equiv bs \pmod{m}$.

(8) (10 Points) Show that for any integer n that $\gcd(n, n + 3)$ is either 1 or 3.