Recent Progress in Mathematics and Engineering on Optimal Graph Labellings with Distance Conditions *

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Abstract

The problem of radio channel assignments with multiple levels of interference can be modelled using graph theory. The theory of integer vertex-labellings of graphs with distance conditions has been investigated for several years now, and the authors recently introduced a new model of real number labellings that is giving deeper insight into the problems. Here we present an overview of the recent outpouring of papers in the engineering literature on such channel assignment problems, with the goal of strengthening connections to applications. Secondly, we present a new contribution to the theory, the formulas for the optimal span of labellings with conditions at distance two for finite complete bipartite graphs.

Dedicated to Prof. Frank K. Hwang on the occasion of his 65th birthday

1 Introduction

For a large network of transmitters spread out in a planar region, the channel assignment problem is to assign a numerical channel, representing a frequency, to each transmitter. The channels assigned to nearby transmitters satisfy some separation constraints so as to avoid interference. The goal is to minimize the portion of the frequency spectrum that must be allocated to the problem, so it is desired to minimize the span of a feasible assignment. In 1980, Hale [31] recast such channel assignment problems in network engineering

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as graph labelling problems. To do this, each transmitter is represented by a vertex, and any pair of vertices that may interfere is represented by an edge in the graph. In the original T-coloring model the labels are integers, and there is a specified set T of forbidden differences between labels for adjacent vertices (corresponding to nearby transmitters).

In the quarter century since Hale's paper appeared, there has been a flurry of activity on both sides: The growth of wireless communication networks has spawned considerable activity in the engineering literature on efficient channel assignments, while mathematicians have been fascinated by the problems of graph vertex-labellings with conditions on their differences. Here, we won't focus on T-coloring (see Roberts [47] (1991) for a nice survey). Instead, we concentrate on labellings with distance constraints.

In 1988 Roberts [48] described a channel assignment that had been proposed to him by Tim Lanfear at NATO. Integer channels are to be assigned to transmitters, and there are two levels of interference to be avoided, depending on the distance between the transmitters, due to spectral spreading: There is some given distance A such that channels for two transmitters within distance A must differ by at least one (that is, be different, since they are integers). This is called the *cochannel constraint*, and the *channel reuse distance* A depends in part on the power of the transmitters. Further, two transmitters that are very close, within distance $A/2$, must differ by at least two, so that the channels cannot be the same or consecutive integers . This is called the *adjacent channel constraint*. The two interference levels are due in part to the spectral spreading of the transmission. Again, the goal is to construct a feasible assignment with minimum span.

Griggs [29] proposed the graph-theoretic analogue of the distance-constrained channel assignment problem, and he generalized it to permit p levels of interference. Specifically, given integers $k_1, \dots, k_p \geq 0$, called *separations*, we say a $L(k_1, k_2, \cdots, k_p)$*-labelling* of a graph G is an assignment of nonnegative integers $f(v)$ to the vertices v of G, such that $|f(u) - f(v)| \geq k_i$ if u and v are at distance i in G. We say that labelling f belongs to the set $L(k_1, k_2, \cdots, k_p)(G)$. We denote by $\lambda(G; k_1, k_2, \cdots, k_p)$ the minimum span over such f, where the span is the difference between the largest and smallest labels $f(v)$. Griggs and Yeh [29] concentrated on the fundamental case of $L(2,1)$-labellings, where the minimum span is denoted more simply by $\lambda(G)$. Many authors have subsequently contributed to the literature on these labellings (see [13, 26, 28, 37, 38, 48]). Increasing attention has been paid recently to more general $L(k_1, k_2)$-labellings (see [10, 11, 12, 22, 23, 24, 35]).

In this paper we endeavor to help bridge the gap between the mathematics/computer science community, which investigates the properties of distance-dependent graph labellings, and the engineering community, which describes the analogous channel assignment problems arising frequently in connection with the evolving technology for communications networks. In Section 2 we call attention to the growing number of papers in the engineering literature on channel assignments. Although they are not all directly related to the lambda-labellings above, these references will likely suggest a wealth of interesting new challenges to the interested reader.

Then in Section 3 we briefly describe the more recently-developed theory of real-number graph labellings introduced by the authors, in which the separations and labels are allowed to range over real numbers. This theory, while it is motivated by the idea that channel frequencies can range over the continuum of real numbers, has exposed properties, even for integer labellings, that escaped us before.

In Section 4, we present a new result, which is the complete determination of the optimal spans of real-number labellings with conditions on the separations at distance two for the family of complete bipartite graphs. Besides contributing to the theory, the techniques illustrated in the proof are of interest.

Finally, we conclude in Section 5 with some directions for future research related to labellings with conditions at distance three or larger.

2 Recent Progress on Channel Assignments

Many applications concern dynamic channel assignment, in which available channels are assigned dynamically in every time slot [30]. In this paper, we focus on fixed channel assignments. The channel assignment problem is usually formulated in the engineering literature as a nonlinear optimization problem. A huge collection of contributions in IEEE journals explore various aspects of current research on radio channel assignment problems, among them "IEEE Transactions on Vehicular Technology" [1, 2, 4, 16, 18, 20, 21, 25, 33, 40, 42, 49, 50, 51], "IEEE Transactions on Wireless Communication" [15, 30], "IEEE/ACM Transactions on Networking" [3, 5], "IEEE/ACM Transactions on Parallel and Distributed Systems" [7, 32], "Wireless Networks" [39, 46, 52], and IEEE Conferences [6, 9, 10, 17, 27, 44].

Wireless networks include cellular mobile networks, wireless computer networks [5], wireless ATM networks [39], and private mobile radio networks [52]. Mobile multimedia applications with variable data rate transmission for such integrated services can be done by means of efficient assignment of time slots, frequencies, codes, or their combinations [15]. Different channel assignment problems in the frequency, time and code domains (with a channel defined as a frequency, a time slot [3], or a control code [5]) can be modeled by graph labelling problem. Ramanathan [46] formulated a framework of channel assignments unified by the similarity of the constraints across these domains.

Efficient utilization of the scarce frequency spectrum for cellular communications is certainly one of the major challenges. Efficient frequency channel assignment will help the modelling and efficient solution of network design problems [41]. Consequently, more users can be supported in the system and this leads to an overall system capacity gain [41].

The processing time required by a channel assignment system that performs optimization by exhaustively searching all combinations increases exponentially relative to the number of base stations (1000 base stations is typical in engineering). As the number of base stations grows, it becomes difficult to find optimal channel assignments in a short time [19]. A rapid growth in demand for wireless communication services has in turn created a need for developing the corresponding efficient channel assignment theory, methods and computational tools.

The frequency channel separations k_i for two transmitters are often inversely proportional to the distance i between them (raising to some constant power $\alpha > 1$) [7]. Most articles assume that the separations are nonincreasing, $k_1 \geq k_2 \geq \ldots \geq k_p$. But this is not required in our mathematics theory. One special case where $k_1 < k_2$ has already arisen: Bertossi and Bonuccelli [5] (1995) introduced an integer "control code" assignment in packet radio networks of computers to avoid hidden terminal interference. This occurs for stations (transmitters), which are outside the hearing range of each other, that transmit to the same receiving stations: It is the $L(0,1)$ graph-labelling problem [36].

3 Optimal Labellings with Real Numbers

Since we can use any frequencies (channels) in the available continuous frequency spectrum, not only from a discrete set, Griggs and Jin [28] extended integer graph labellings to allow the labels and separations k_i to be nonnegative *real* numbers. We use the same notation as before, $L(k_1,\ldots,k_p)(G)$ and $\lambda(G;k_1,\ldots,k_p)$, but now the span of a real labelling is the difference between the supremum and the infimum of the labels used, and λ is the infimum of the spans of such labellings. For example, $\lambda(P_4;\sqrt{2},1) = \sqrt{2}+1$ for path P_4 on four vertices

For graphs of bounded maximum degree, Griggs and Jin proved the existence of an optimal labelling of a nice form, in which all labels belong to the discrete set, denoted by $D(k_1,k_2,\ldots,k_p)$, of linear combinations $\sum_i a_i k_i$, with nonnegative integer coefficients a_i.

Theorem 3.1 (The D-Set Theorem [28]). *Let G be a graph, possibly infinite, with finite maximum degree. Let real numbers $k_i \geq 0$, $i = 1,2,\ldots,p$. Then there exists a finite optimal $L(k_1,k_2,\ldots,k_p)$-labelling $f^* : V(G) \to [0,\infty)$ in which the smallest label is 0 and all labels belong to the set $D(k_1,k_2,\ldots,k_p)$. Hence, $\lambda(G;k_1,k_2,\ldots,k_p)$ belongs to $D(k_1,k_2,\ldots,k_p)$.*

Some natural properties of distance-constrained labellings become more evident in the setting of real number labellings. Due to the D-set Theorem, previous optimal integer labelling results are compatible with our optimal real number labelling results. In particular, we observe the following Scaling property: For real numbers $d, k_i \geq 0$, $i = 1,2,\ldots,p$,

$$\lambda(G; d\cdot k_1, d\cdot k_2,\ldots, d\cdot k_p) = d\cdot\lambda(G;k_1,k_2,\ldots,k_p).$$

In [28, 35], we proved $\lambda(G;k_1,k_2,\ldots,k_p)$ is a continuous function of the separations k_i for any graph G with finite maximum degree. Hence, results about the minimum spans $\lambda(G;k_1,k_2,\ldots,k_p)$ for k_i being rational numbers can often be extended into the results for k_i being real numbers. Indeed, by Scaling, it is usually enough to obtain results for integer k_i. But the analysis is more clear, and more results emerged, by considering real number labellings.

For any fixed p and any graph G with finite maximum degree, we [28] proved that $\lambda(G;k_1,k_2,\ldots,k_p)$ is a piecewise-linear function of real numbers k_i if G is finite or if $p = 2$.

By Scaling, we have that for $k_2 > 0$, $\lambda(G,k_1,k_2) = k_2\lambda(G;k,1)$, where $k = k_1/k_2$. This reduces the two-parameter function to a one parameter function, $\lambda(G;k,1)$, $k \geq 0$. As just discussed, we can be sure it is a continuous, nondecreasing, piecewise-linear function with finitely many pieces. Further, each piece has the form $ak+b$ for some integers

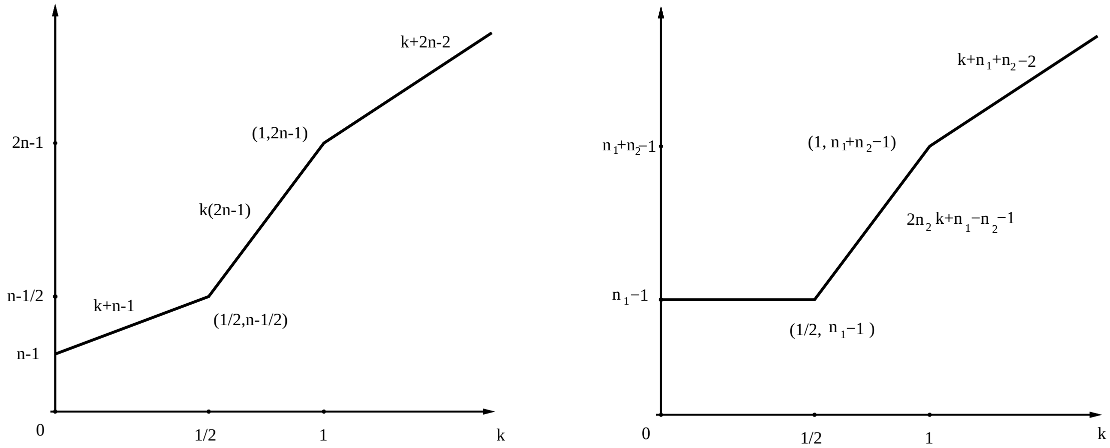

Figure 1: Minimum Spans $\lambda(K_{n,n};k,1)$ (left) and $\lambda(K_{n_1,n_2};k,1), n_1 > n_2$ (right)

$a, b \geq 0$. We prove upper bounds on the minimum span by labelling constructions, and lower bounds by restricting our focus on some induced subgraph H of G, since a lower bound on H is a lower bound on G.

4 Optimal Spans of Complete Bipartite Graphs.

We have the following minimum spans for complete bipartite graphs (see Figure 1).

Theorem 4.1. *If $n_1 \geq n_2$ are positive integers and k is a non-negative real number, then*

$$\lambda(K_{n_1,n_2};k,1) = \begin{cases} \max\{n_1-1, n_2-1+k\}, & \textit{if} \quad 0 \leq k \leq \frac{1}{2}; \\ (2n_2-1)k + \max\{n_1-n_2-1+k, 0\}, & \textit{if} \quad \frac{1}{2} \leq k \leq 1; \\ k+n_1+n_2-2, & \textit{if} \quad k \geq 1. \end{cases}$$

Proof: Let $G = K_{n_1,n_2}$ have partite sets $X = \{x_0, x_1, \ldots, x_{n_1-1}\}$ and $Y = \{y_0, y_1, \ldots, y_{n_2-1}\}$. We first present labellings that achieve the stated spans.

- For $0 \leq k \leq \frac{1}{2}$, let $f(x_i) = i$ when $0 \leq i \leq n_1 - 1$, and let $f(y_j) = j + k$ when $0 \leq j \leq n_2 - 1$.
- For $\frac{1}{2} \leq k \leq 1$, let $f(x_i) = 2ik$ when $0 \leq i \leq n_2 - 1$ and $f(x_i) = 2n_2k + i - n_2$ when $n_2 \leq i \leq n_1 - 1$, and let $f(y_j) = (2j+1)k$ when $0 \leq j \leq n_2 - 1$.
- For $k \geq 1$, let $f(x_i) = i$ when $0 \leq i \leq n_1 - 1$, and $f(y_j) = n_1 - 1 + j + k$ when $0 \leq j \leq n_2 - 1$.

Next we derive the lower bounds according to the following cases. Suppose f is an optimal labelling for G.

Case 1: $0 \leq k \leq \frac{1}{2}$. By the separation conditions, there is at most one label $f(v)$ for $v \in X$ in each interval $[i, i+1)$. So X has at least one label beyond $[0, n_1 - 1)$, and the same is true for Y when $n_1 = n_2$. Consequently, the span is at least $n_1 - 1$, and is at least $n_1 - 1 + k = n_2 - 1 + k$ for $n_1 = n_2$, as desired.

Case 2: $k \geq \frac{1}{2}$. List the $n_1 + n_2$ labels $f(v)$ in increasing order. Note that two consecutive labels in the ordering differ by at least k (respectively, at least 1), if they are used for vertices in opposite (resp., the same) partite sets of G. Let a (resp., b) denote the number of pairs of consecutive labels for vertices in opposite (resp., the same) partite sets. Hence, $a + b = n_1 + n_2 - 1$, and the span of f is at least $ak + b$. No matter what order the labels are in, it must be that $a \geq 1$ and $b \geq \max\{n_1 - n_2 - 1, 0\}$. For the case of $\frac{1}{2} \leq k \leq 1$, the span is at least $ak + b \geq (n_1 + n_2 - 1 - \max\{n_1 - n_2 - 1, 0\})k + \max\{n_1 - n_2 - 1, 0\} = (2n_2 - 1)k + \max\{n_1 - n_2 - 1 + k, 0\}$, as desired. For the case of $k \geq 1$, the span is at least $ak + b \geq k + (n_1 + n_2 - 1 - 1) = k + n_1 + n_2 - 2$, as desired. $\square$

Independently of us, Calamoneri, Pelc, Petreschi solved the special case of stars:

Corollary 4.2 ([11]). *For real number $k \geq 0$, and integer $n \geq 2$, we have*

$$\lambda(K_{n,1}; k, 1) = \begin{cases} n - 1 & \text{if } 0 \leq k \leq \frac{1}{2} \\ 2k + n - 2 & \text{if } \frac{1}{2} \leq k \leq 1 \\ k + n - 1 & \text{if } k \geq 1 \end{cases}$$

5 Future Research Directions

We conclude by describing some of the directions for future research which have applications in wireless communications. Rapid growth in the demand for mobile communication has led to intense research and development efforts towards a new generation of cellular systems. Due to the decreasing cost of transmitter constructions, a large number of small cells is expected in the new generation of wireless systems [7]. Small cell systems allow greater channel reuse and large capacity [45]. When each cell has significant power, the channel reuse distance will be larger, and it means we need to consider $L(k_1, k_2, \ldots, k_p)$ for larger $p \geq 3$.

There are several papers in engineering with research on the case that $k_1 = k \geq k_2 = k_3 = \cdots = k_p = 1$. Van den Heuvel, Leese, and Shepherd [34] show a result which is equivalent to $\lambda(P_n; 2, 1, 1) = 4$, for a path P_n, $n \geq 2$. Bertossi, Pinotti, Tan [7] give values and labellings for the triangular lattice $\Gamma_\triangle$ (which is the 6-regular, infinite planar lattice): $\lambda(\Gamma_\triangle; 1, 1, 1)$ and $\lambda(\Gamma_\triangle; 2, 1, 1)$. Bertossi et al. [7] and then Panda et al. [44] present a lower bound for the square lattice $\Gamma_\square$ (the 4-regular planar lattice) independently, $\lambda(\Gamma_\square; 1, 1, \ldots, 1) \geq \lfloor \frac{p^2+2p}{2} \rfloor$, where p is as above and hence $p + 1$ is the channel reuse distance.

Motivated by engineering problem in communications, Chartrand et al. [14] introduce "radio k-coloring" on the vertices of a graph G, with $x \in V(G)$ labelled $c(x)$ such that $d(u, v) + |c(u) - c(v)| \geq k + 1$, where k is a fixed integer between 1 and the diameter of G. When k is the diameter of G, and G is finite, it is called radio labelling or radio coloring. Subsequently, many papers (see [14, 8, 53]) contributed to the radio labelling problems in mathematics. It is interesting to consider the corresponding $L(k, k - 1, k - 2, \cdots, 2, 1)$-labellings of a (possibly infinite) graph G for any fixed number k (instead of the diameter).

We believe that the minimum span $\lambda(G; k, k-1, k-2, \cdots, 2, 1)$ is a piecewise-polynomial function of k.

Moon et al. [43] mention a mobile network using a frequency hopping technique (called synthesizer hopping). In the stage of network design, a list of channels is assigned to each transmitter in the network under some separation conditions. In the usage stage of the system, the traffic carriers (i.e., transmitted signals) hop in a defined sequence over a predefined set of frequencies of the transmitters. So we assign a pre-generated channel list for each transmitter in the design stage. Then assign each traffic carrier a channel at the transmitter among the corresponding channel list synchronically (dynamically) in the system usage stage. For example, some of the transmitters (like military and governmental stations) already have preassigned labels corresponding to frequency channels which are not allowed to change [8]. That is, assign all vertices label-lists of fixed size, then we may assign each traffic carrier at each vertex a label in its label list (called list labelling). During the assignment, we may try to equalize the number of cells using the same channel [45] as much as possible.

Concerning graph models of wireless networks, Dubhashi et al. [17] present bounds on the minimum span for $L(2, 1, 1, \cdots, 1)$-labelling of the d-dimensional square lattice (grid), in which $V(G) = \mathbb{Z}^p$, and two vertices, say $(x_1, x_2, \ldots, x_p)$ and $(y_1, y_2, \ldots, y_p)$, are joined by an edge whenever $\sum_{i=1}^{p} |x_i - y_i| = 1$. The motivation is that when the networks of several service providers overlap geographically, they must use different channels for their clients. The overall network can be modelled in a higher dimensional lattice.

We expect that by extending these problems of optimal integer graph labelling to more general real number labellings, our developing theory will give more insight into the original problems.

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