

## Syllabus Analysis Qualifying Exam

These are the topics to be covered for preparation for the Qualifying Exam portion in Analysis. It is generally expected that Instructors in Math 703-704 will cover the majority of these topics in some detail.

### Lebesgue theory of Measure and Integration

Metric spaces, compactness, continuous functions and Weierstrass approximation theorem.

Outer measure, measurable sets, measure spaces, complete and regular measures.

Integration, Fatou's lemma and convergence theorems

The Extension Theorem, product measure and Fubini's theorem,

Lebesgue-Stieltjes integral

Absolute continuity, Vitali's lemma, differentiation theory for monotone functions and integrals, functions of bounded variation.

Egorov and Lusin Theorems

Definition of  $L^p$ , Hölder and Minkowski inequalities, completeness of  $L^p$ , approximation by step and continuous functions.

### Complex Analysis

Analytic Functions: complex derivatives and Cauchy-Riemann equations, analyticity, special functions:  $\log(z)$ ,  $e^z$ , trig functions.

Conformal mappings and linear fractional transformations.

Line integrals, Cauchy's theorem and its consequences:

Cauchy integral formula, maximum modulus, power series,

Fundamental Theorem of Algebra.

Classification of zeros and singularities, Laurent series,

Argument principle.

Residue theorem, evaluation of integrals and series.

### **Reference Texts:**

0. Elias M. Stein & Rami Shakarchi, *Real Analysis*, Princeton Lectures in Analysis II, 2005
1. W. Ruckle, *Modern Analysis*, PSW-Kent, Boston, 1991
2. W. Rudin, *Real and Complex Analysis* (2nd edition), McGraw-Hill, New York, 1974.
3. H. Royden, *Real Analysis* (4th edition), Macmillan Co., New York.
4. G. Folland, *Real Analysis*, John Wiley & Sons, New York, 1984.
5. L. Ahlfors, *Complex Analysis* (3rd edition), McGraw-Hill, New-York, 1979.
6. J. Conway, *Functions of a Complex Variable*, Springer-Verlag, New York, 1978.
7. Saks and Zygmund, *Analytic Functions*, Elsevier, New York, 1971.