Set Operations with two sets

Defs. **Definitions**. Let A_1 and A_2 be subsets of a universal set U. ⟨eg, $U = \mathbb{R}^2$ & draw 2 subsets A_1 and A_2 in $\mathbb{R}^2$⟩

1. The **relative complement of** A_1 **with respect to** A_2, also called A_2 **set minus** A_1, is the set

$$A_2 \setminus A_1 \overset{\text{def}}{=} \{x \in U : x \in A_2 \ \ \underset{\sim}{\text{and}} \ \ x \notin A_1\} \underset{\text{notation}}{\overset{\text{other}}{=}} A_2 - A_1.$$

2. The **complement** of A_1, denoted $(A_1)^C$, is the set of all elements of U that are not in A_1. So

$$(A_1)^C \overset{\text{def}}{=} \{x \in U : x \notin A_1\} \overset{\text{note}}{=} U \setminus A_1 \overset{\text{note}}{=} U - A_1$$

3. The **union** of A_1 and A_2, denoted $A_1 \cup A_2$, is the set of all elements that are in A_1 or A_2. Thus

$$A_1 \cup A_2 \overset{\text{def}}{=} \{x \in U : x \in A_1 \ \ \text{or} \ \ x \in A_2\} \underset{\text{notation}}{\overset{\text{other}}{=}} \bigcup_{i=1}^{2} A_i$$

$$\overset{\text{note}}{=} \{x \in U : \text{there exists an } i \in \{1,2\} \text{ such that } x \in A_i\} \underset{\text{notation}}{\overset{\text{other}}{=}} \bigcup_{i \in \{1,2\}} A_i$$

4. The **intersection** of A_1 and A_2, denoted $A_1 \cap A_2$, is the set of all elements in both A_1 and A_2. So

$$A_1 \cap A_2 \overset{\text{def}}{=} \{x \in U : x \in A_1 \ \ \text{and} \ \ x \in A_2\} \underset{\text{notation}}{\overset{\text{other}}{=}} \bigcap_{i=1}^{2} A_i$$

$$\overset{\text{note}}{=} \{x \in U : \text{for all } i \in \{1,2\} \text{ we have that } x \in A_i\} \underset{\text{notation}}{\overset{\text{other}}{=}} \bigcap_{i \in \{1,2\}} A_i$$

Union and Intersection over arbitrary index set

Above we had 2 subsets (A_1 and A_2) of a universe U and our index set I was $I = \{1,2\}$. We can take the union and intersection of 17 sets (so our indexing set I would be $I = \{1, 2, \ldots, 17\}$).

Defs. **Definitions**. ⟨Warning. Do not mix up the indexing set I and universe U.⟩
Let I be a set ⟨we call I the *index set*⟩. Let U be a universe ⟨where the sets A_i live⟩
Consider a family ⟨i.e., collection⟩ of subsets $\{A_i : i \in I\}$ of U ⟨so $A_i \subseteq U$ for each $i \in I$⟩.

5. The **union** of family/collection of subsets $\{A_i : i \in I\}$ over the index set I is:

$$\bigcup_{i \in I} A_i \overset{\text{def.}}{=} \{x \in U : \text{there exists an } i \in I \text{ such that } x \in A_i\}$$

$$\overset{\text{i.e.}}{=} \{x \in U : x \text{ is in (at least) one of the } A_i\text{'s}\} \overset{\text{note}}{\subseteq} U.$$

6. The intersection of family/collection of subsets $\{A_i : i \in I\}$ over the index set I is:

$$\bigcap_{i \in I} A_i \overset{\text{def.}}{=} \{x \in U : \text{for all } i \in I \text{ we have that } x \in A_i\}$$

$$\overset{\text{i.e.}}{=} \{x \in U : x \text{ is in all of the } A_i\text{'s}\}$$

$$\overset{\text{i.e.}}{=} \{x \in U : \text{ if } i \in I \text{ then } x \in A_i\} \overset{\text{note}}{\subseteq} U.$$

So:.

$$\left[x \in \bigcup_{i \in I} A_i \right] \Leftrightarrow \left[(\exists i \in I) \ [x \in A_i] \right] \quad \text{while} \quad \left[x \in \bigcap_{i \in I} A_i \right] \Leftrightarrow \left[(\forall i \in I) \ [x \in A_i] \right]$$

Note when $I = \emptyset$

$$\bigcup_{i \in \emptyset} A_i = \emptyset \qquad \text{while} \qquad \bigcap_{i \in \emptyset} A_i = U.$$