**Book's ER 3.4.17**

§3.4
BS4p84–85

Alternate the terms of the sequences $\left\{1+\frac{1}{n}\right\}$ and $\left\{-\frac{1}{n}\right\}$ to obtain the sequence $\{x_n\}$ given by

$$\left\{2, -1, \frac{3}{2}, -\frac{1}{2}, \frac{4}{3}, -\frac{1}{3}, \frac{5}{4}, -\frac{1}{4}, \ldots\right\}.$$

Determine the values of the $\limsup x_n$ and $\liminf x_n$. Also find $\sup\{x_n\}$ and $\inf\{x_n\}$.

Added remark. If you explain your answer intuitively, then a formal proof is not needed.

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**HINT to get you started.**

We have a positive-termed sequence $P = \{p_n\}_{n\in\mathbb{N}}$ and a negative-termed sequence $Q = \{q_n\}_{n\in\mathbb{N}}$ where

$$p_n \overset{\text{def}}{=} 1 + \frac{1}{n} \overset{\text{note}}{>} 0 \qquad \text{and} \qquad q_n \overset{\text{def}}{=} -\frac{1}{n} \overset{\text{note}}{<} 0$$

and "shuffle" these 2 sequences to get the sequence

$$X = \{x_n\}_{n\in\mathbb{N}} = \{p_1, q_1, p_2, q_2, p_3, q_3, p_4, q_4, \ldots\}.$$

Recall the tails of any sequence $Z = \{z_n\}_{n\in\mathbb{N}}$ are the sets

$$T_n^Z = \{z_n, z_{1+n}, z_{2+n}, \ldots\} = \{z_k \colon k \geq n\}$$

and

$$\limsup_{n\to\infty} z_n \overset{\text{def}}{=} \lim_{n\to\infty} \left[\sup T_n^Z\right]$$

$$\liminf_{n\to\infty} z_n \overset{\text{def}}{=} \lim_{n\to\infty} \left[\inf T_n^Z\right]$$