Remark: ER 3.2.51, ER 3.2.52, ER 3.2.53 go together.

In ER 3.2.52 be careful not to make a mistake similar to the mistake in ER 3.2.51.

**ER 3.2.52** §3.2 BS4p69-70

Let the sequence $\{x_n\}_{n=1}^{\infty}$ converges to $x_0$. Let $\{a_n\}_{n=1}^{\infty}$ be the averages of the $x_n$'s, i.e.,

$$a_n \;=\; \frac{x_1 + x_2 + \cdots + x_n}{n} \quad \text{for each} \quad n \in \mathbb{N}.$$

Prove that $\{a_n\}_{n=1}^{\infty}$ converges to $x_0$.

HINTS. This is an $\frac{\varepsilon}{2}$-argument. First note $x_0 = \sum_{i=1}^{n} \frac{x_0}{n}$. So if $1 \le I \le n$, then

$$\begin{aligned}
|x_0 - a_n| &= \left| \sum_{i=1}^{n} \frac{x_0}{n} - \sum_{i=1}^{n} \frac{x_i}{n} \right| \\
&= \left| \sum_{i=1}^{n} \frac{x_0 - x_i}{n} \right| \\
&\overset{\triangle-\text{ineq.}}{\le} \sum_{i=1}^{n} \left| \frac{x_0 - x_i}{n} \right| \\
&= \sum_{i=1}^{n} \frac{|x_0 - x_i|}{n} \\
&= \sum_{i=1}^{I} \frac{|x_0 - x_i|}{n} + \sum_{i=I+1}^{n} \frac{|x_0 - x_i|}{n}
\end{aligned}$$

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