On this homework ⟨provided your specifically reference it when used⟩ you may use the following Theorem from class.

Nested Interval Property. ⟨Really a Theorem⟩.

Let $(I_n)_{n=1}^{\infty}$ be a sequence of nonempty closed bounded intervals of $\mathbb{R}$ that are nested in the sense

$$I_{j+1} \subseteq I_j \quad \text{for each } j \in \mathbb{N}.$$

Then

(1) $\bigcap_{n=1}^{\infty} I_n$ is nonempty,

(2) and if futhermore the $\lim_{n\to\infty} \text{length}\,(I_n) = 0$ then $\bigcap_{n=1}^{\infty} I_n$ has preciously one element.

Variant of book's ER 2.5.7+2.5.8

§2.5
BS4p52

1. For each $n \in \mathbb{N}$, let

$$I_n \ := \ \left[0, \frac{1}{n}\right].$$

Prove that $\bigcap_{n=1}^{\infty} I_n = \{0\}$.

2. For each $n \in \mathbb{N}$, let

$$J_n \ := \ \left(0, \frac{1}{n}\right).$$

Prove that $\bigcap_{n=1}^{\infty} J_n = \emptyset$.

3. In the above statement of the **Nested Interval Property**, can the assumption that the intervals are closed be omitted? Justify your answer.

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