

On this homework, specifically say where you are using Archimedean's Property (or one of it's corollaries) when you use it. Below is a list of the versions of Archimedean's Property that we showed in class.

**Thm.** (Archimedean's Property)  $(\forall b \in \mathbb{R}) (\forall a \in \mathbb{R}^{>0}) (\exists n \in \mathbb{N}) [b < na]$

**Cor. 1.**  $(\forall x \in \mathbb{R}) (\exists n \in \mathbb{N}) [x < n]$

**Cor. 2.**  $(\forall \varepsilon > 0) (\exists n \in \mathbb{N}) \left[\frac{1}{n} < \varepsilon\right]$

**Cor. 3.**  $(\forall z \in \mathbb{R}^{>0}) (\exists n \in \mathbb{N}) [n - 1 \leq z < n]$

On this homework, you may use the below lemma provided you prove the lemma using induction.

**Lemma 1.** If  $n \in \mathbb{N}$ , then  $n < 2^n$ . (Note this lemma is ER 1.2.13 p. 16.)

**Variant of book's ER 2.4.14**

§2.4  
BS4p46

Let  $\varepsilon > 0$ . Prove that there exists  $n \in \mathbb{N}$  such that

$$\frac{1}{2^n} < \varepsilon.$$

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