Variant of book's ER 2.4.8

§2.4
BS4p45

Let X be a nonempty set. Let the function $f\colon X \to \mathbb{R}$ and $g\colon X \to \mathbb{R}$ have bounded ranges.

Prove

$$\sup\{f(x) + g(x)\colon x \in X\} \;\leq\; \sup\{f(x)\colon x \in X\} \;+\; \sup\{g(x)\colon x \in X\}. \tag{1}$$

Also give an example showing that the inequality in (1) can not be an equality.

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