

**Def.** For  $r \in \mathbb{R}$  and  $B \subseteq \mathbb{R}$ , define  $rB := \{rb : b \in B\}$ .

In class and on pervious HW, we showed the following two theorems.

**Thm 1.** Let  $T$  be a nonempty bounded subset of  $\mathbb{R}$ . Let  $p > 0$ . Then  $\sup(pT) = p \sup T$ .

**Thm 2.** Let  $T$  be a nonempty bounded subset of  $\mathbb{R}$ . Let  $p > 0$ . Then  $\inf(pT) = p \inf T$ .

**Thm 3.** Let  $T$  be a nonempty bounded subset of  $\mathbb{R}$ . Then  $\sup(-T) = -\inf T$ .

**Variant of book's ER 2.4.4b**

§2.4  
BS4p45

**Thm 4.** Let  $T$  be a nonempty bounded subset of  $\mathbb{R}$ . Let  $n < 0$ . Then  $\sup(nT) = n \inf T$ .

**Thm 5.** Let  $T$  be a nonempty bounded subset of  $\mathbb{R}$ . Let  $n < 0$ . Then  $\inf(nT) = n \sup T$ .

1. Give a (short) proof of Thm 4 by using (some of): Thm 1, Thm 2, Thm 3.
  2. Give a (short) proof of Thm 5 by using (some of): Thm 1, Thm 2, Thm 3, Thm 4.
- . HINT:  $nT = (-n)(-T)$ .
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