ER 2.2.51 **Four Triangle Inequalities.**

§2.2
BS4p32

The following facts are proved in the book.

Three Triangle Inequalities. Let $x, y \in \mathbb{R}$. Then

$$|x+y| \le |x| + |y| \qquad \text{(Thm. 2.2.3)}$$

$$|x-y| \le |x| + |y| \qquad \text{(Cor. 2.2.4b)}$$

$$\big|\, |x| - |y| \,\big| \le |x-y| \qquad \text{(Cor. 2.2.4a)}$$

Using the above 3 triangle inequalities ⟨which are proved in the book⟩, prove ⟨a fourth triangle inequality⟩

$$\big|\, |x| - |y| \,\big| \le |x+y| \tag{4}$$

Remark. The purpose of this problem is to get all four Triangle Inequalities; indeed, **after** proving the inequality in (4), we can combine (4) with (Thm. 2.2.3), (Cor. 2.2.4a), and (Cor. 2.2.4b) to get the four inequalites:

$$\boxed{\big|\, |x| - |y| \,\big| \le |x \pm y| \le |x| + |y|} \,. \qquad (\triangle\text{-inequalities})$$

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